2023

AN ECONOMETRIC MODEL OF THE US GOVERNMENT YIELD CURVE LEVELS AND DYNAMICS

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AN ECONOMETRIC MODEL OF THE US GOVERNMENT YIELD CURVE

LEVELS AND DYNAMICS

A thesis submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE

to the faculty of the

DEPARTMENT OF BUSINESS ANALYTICS

of

THE PETER J. TOBIN COLLEGE OF BUSINESS

at

ST. JOHN'S UNIVERSITY
New York
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Date Submitted 12/9/2022 Date Approved 12/9/2022

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ABSTRACT

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Katerina Yiasoumi

The yield curve is the graph of the relationship of the nominal yield to maturity (ytm) on bonds of a similar asset class to different bond maturities at a point in time. Yield curves exist for every sector of the fixed income asset class, e.g., corporate, municipal, emerging markets, high yield, etc. not only for the United States, but for all bond markets worldwide. This research uses graphical techniques, descriptive statistics, correlation, as well as linear and non-linear regression to model each of nine treasury bills, notes, and bond ytms along the US Government bond maturity spectrum. The result is a system of equations that explain and predict the US Government yield curve levels, slopes and dynamics, a phenomenon largely responsible for driving the value of US$300 Trillion of global debt. The research results are highly relevant to institutional and retail investors, borrowers, and lenders, as well as academics and policy makers.

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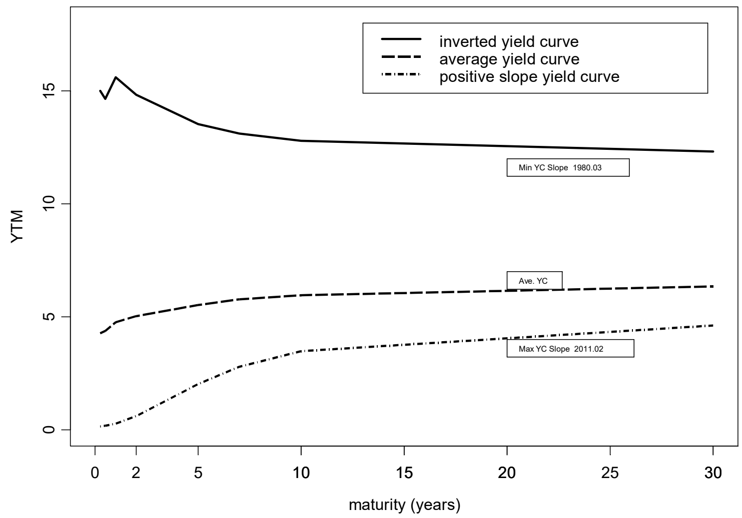
CHAPTER 1: INTRODUCTION

1.1 Introduction

The Yield to Maturity (YTM) is the internal rate of return for a bond assuming that the asset is held until its maturity date. It is the interest rate which equates investment value to the present value of the cash flow. A bond's yield to maturity rises or falls depending on its market value and how many payments remain to be made. The yield to maturity is important, as it enables investors to compare different securities and the expected returns from each, in order to construct their own portfolio.

The yield curve is the graph of the relationship of the nominal yield to maturity (YTM) on bonds of a similar asset class to different bond maturities at a point in time. Yield curves exist for every sector of the fixed income asset class, e.g., corporate, municipal, emerging markets, high yield, etc. not only for the United States, but for all bond markets worldwide. Generally, yield curves have a positive slope, sometimes negative (inverted) or sometimes inverted parabolic (humped) and they are constantly shifting and pivoting up and down.

Figure 1 Three Yield Curve Shapes



The yield curve is largely responsible for determining the value of US$300 Trillion of global debt and it is alleged to give predictions for the future path of the economy. A positive yield curve slope predicts economic growth and negative curve slope predicts recession. The current yield curve is inverted, something that indicates future economic recession.

There are three economic theories that attempt to explain the Yield Curve shape; The Liquidity Preference Theory which hypothesizes that the lenders prefer to lend short term and they demand higher interest rates to lend longer term. Fisher Effect Theory which hypothesizes that as inflation increases so do the nominal interest rates. Preferred Habitat hypothesizes that the Yield curve reflects supply and demand in preferred investment maturity ranges.

The focus of this research is the US Government Bond Sector which consists of nine US Government Debt Instruments. Treasury Bills have 3-, 6 -,9- or 12-month original maturity, Treasury Notes have 2–10-year original maturity and Treasury Bonds which have 20- or 30-year original maturity.

This research uses graphical techniques, descriptive statistics, correlation, as well as multiple linear and nonlinear regression to model each of the nine treasury bills, notes, and bonds yield to maturities along the US Government bond maturity spectrum. The result is a system of nine equations that explains and predicts the US Government yield curve levels, slopes, and dynamics. The research results are highly relevant to institutional and retail investors, borrowers, and lenders, as well as academics and policy makers.

For this research 412 monthly observations were gathered from the Federal Reserve Bank of St. Louis database and nine regression models were created to model nine Yield to Maturities (3 ,6,12-month Treasury Bills, 2–10-year Treasury Notes and 20, 30-year Treasury Bonds). These models were used to estimate the sensitivity of each Yield to Maturity to inflation, money supply and Consumer Price Index, and identify the "Real" Yield curve, i.e., the yield curve where the inflation is equal to zero. This research produces evidence supporting the following economic theories, Liquidity Preference Theory, Fisher Effect Hypothesis and Negative Demand for real money balances.

1.2 Structure of the Thesis

This thesis is divided into 6 chapters.

1. Chapter 2 provides definitions and background for economic terms and variables that were used for this research. This chapter aims to discuss concepts such as the yield curve, nominal and real interest rates, the term structure of interest rates, Consumer Price Index, Inflation, Money Supply, and the Federal Reserve System.
2. Chapter 3 discusses the relationships between money supply, cpi and inflation with the interest rates and how they are connected to several economic theories such as: liquidity preference theory, fisher effect theory and negative demand for money. Furthermore, previous research studies, empirical evidence and conclusions regarding the interest rates predictions and relations with money supply and inflation are provided.
3. Chapter 4 describes the dataset and the variables that were used in this paper by performing a preliminary analysis. Histograms, time-series plots, scatter plots, descriptive statistics and the correlation matrix of the variables used are provided and explained.

4. Chapter 5 continues with the regression analysis. It is investigated which model will be the optimal to use to predict the US Government Yield Curve and provides evidence that will support or contradict several economic theories.
5. Chapter 6 includes the summary, conclusions, and suggestions for further research.

CHAPTER 2: ECONOMIC BACKROUND AND DEFINITIONS

2.1 Yield Curve

The yield curve is the graph of the relationship of the nominal yield to maturity (YTM) on bonds having equal credit quality with different maturity dates at a point in time. The yield curve is largely responsible for driving the value of US$300 Trillion of global debt and gives predictions for the future path of the economy. Yield curves exist for every sector of the fixed income asset class, e.g., corporate, municipal, emerging markets, high yield, etc. not only for the United States, but for all bond markets worldwide.

Generally, yield curves have a positive slope, sometimes negative (inverted) or sometimes inverted parabolic (humped). They are constantly shifting and pivoting up and down. Most of the time the yield curve has a positive slope where interest rates increase with maturity. Theories, such as Expectations Theory, Liquidity Preference Theory and Inflation Premium Theory, have been developed to explain this phenomenon. Moreover, in some rarely cases, the values of short-term interest rates may exceed the values of long-term interest rate having as a result the inverted yield curve or in other words a yield curve with a negative slope. Several reasons could lead to this result. The inverted shape of the yield curve can be attributed to the Federal Reserve Board's high influence towards the bond market. In occasions where the aim is to control the possibly high inflation or remove excess liquidity from the economy, the Fed may increase the short-term interest rates to the point that they exceed the values of the long-term interest rates and cause the inverted yield curve shape. On the other hand, long term interest rates are highly influenced by supply and demand in the bond market. They may significantly decrease due to lower inflation rate expectations or economic recession expectations; the opposite could be said about significant increases. Flat yields curves can also be encountered. In this case, usually there is no dramatic change between the interest rates of bonds with different maturities at a specific point in time. This could occur due to an economic stability i.e., no significant changes in the economy, investment market or future inflation.

2.2 Term Structure of Interest Rates

The term structure of interest rates is the relationship of interest rates of zero-coupon bonds and different maturity dates. Any bond can be viewed as a package of zero-coupon instruments and the bond value should be equal to the value of all the constituent zero-coupon instruments. The spot rate which is the yield to maturity of zero-coupon Treasury instruments will be used to calculate the value of each zero-coupon instrument with the same maturity date. However, zero coupon treasury instruments with maturity higher than one year do not exist, which makes it difficult to calculate the spot rates by observing the behavior of treasury securities in the bond market. Nevertheless, a spot rate yield curve can theoretically be derived from the yield curve and determine the values of the spot rates. This theoretical spot rate yield curve is considered to be the graphical representation of the term structure of the interest rates.

2.3 Term Structure of Forward Rates

The Term Structure of Forward Rates is the relationship between the forward rates of bonds with the maturity dates. Forward rates are calculated from the spot rates, and they represent the market's consensus of future interest rates. By estimating the future interest rates, investors can arrange their investment plan and decide whether keeping a bond investment until maturity or investing in shorter-term bond and reinvesting when the shorter-term bond matures will be more profitable.

2.4 Nominal and Real Interest Rates

An interest rate is the internal rate of return on a bond investment, and it is expressed as a percentage of the total amount of the investment.

2.4.1 Nominal Interest Rate

The nominal interest rate is the rate of an investment stated by banks, debt issuers, and investment firms and it is expressed as the sum of the real interest rate and the expected inflation. Central banks determine short-term nominal interest rates to slow down or increase the economic activity in the market by making the interest rates issued by banks and other institutions less or more attractive to investors.

2.4.2 Real Interest Rate

The real interest rate is the actual rate of return received by an investment. It is calculated by subtracting the inflation rate from the nominal interest rate. The Fisher Effect Theory, which was developed by the economist Irving Fisher, supports that if inflation expectations rise more than the nominal interest rate increases, the real interest rate will decrease.

$$Real\ Interest\ Rate = Nominal\ Interest\ Rate - Inflation\ Expectations\ Rate$$

2.5 CPI, Inflation and GDP

2.5.1 Consumer Price Index (CPI)

The consumer price index (CPI) measures the average price changes of goods and services bought by a typical consumer. The Bureau of Labor Statistics (BLS), which is part of the Department of Labor, computes and reports the CPI on a monthly basis. The cost of living of a typical consumer can be divided into various categories of goods and services such as housing, transportation, food and beverages, medical care, education and communication, recreation, apparel and other goods and services. To measure the CPI, the consumer's basket of goods and services of the current year are identified, and its costs are computed. The BLS calculates the cost of living by also taking into consideration the importance of each category's price for a typical consumer. For example, if the consumption of product A is higher than the consumption of product B, then a greater weight should be given to the price of product A to measure the CPI. After computing the cost of the basket of goods in each year, one year should be chosen as the benchmark against which other years are to be compared. The Consumer Price Index can be calculated by the following formula:

$$CPI = \frac{Price\ of\ basket\ of\ goods\ and\ services\ in\ current\ year}{Price\ of\ basket\ in\ base\ year} * 100$$

2.5.1.a Problems in Measuring the Cost of Living

There are three problems that may be observed while measuring the CPI. First, it is true that the prices of goods through the years don't always change proportionately. Consumers, in this case, prefer to buy goods that their prices haven't radically changed or have become less expensive. If this phenomenon of consumer substitution is not taken into consideration and the CPI is calculated on a fixed basket of goods, the index will measure a much larger increase in the average prices of goods and services than consumers are actually experiencing. This problem is called the substitution bias. A second problem that can arise with CPI is the introduction of new goods. As new goods are introduced in the consumer market, consumers have a greater variety of goods from which to choose, something that will increase the dollar value. On the other hand, when goods are removed from the market or being replaced by a single product the cost-of-living decreases. As CPI is based on a fixed basket of goods and services, the index does not reflect this increase or decrease of the dollar value. The last problem regarding the CPI is called the unmeasured quality change. This problem can occur when the quality of a good aggravates or improves through the years, as this quality change has an impact on the dollar value. Despite the efforts of the BLS to avoid this problem by adjusting the prices of goods according to their changes in quality, the problem has not been resolved due to the difficulty of measuring quality.

2.5.1.b Comparing dollar figures from different times

The CPI can be used to compare dollar figures from different points in times. As the cost-of-living changes over time the only way to compare the salaries of the past with the salaries of the present is by knowing the level of prices in two points of time (present and past). This can be achieved with the following formula, which turns an amount of dollars in the past (T) into today's amount of dollars:

$$Amount\ in\ today's\ dollars = Amount\ in\ year\ T * \frac{Price\ level\ today}{Price\ level\ in\ year\ T}$$

2.5.2 Inflation

Inflation is the variable that describes the rise of the overall level of prices in the economy. This phenomenon can be calculated by the inflation rate, which is the percentage change in the prices level from the previous period. Inflation can be measured either from the GDP deflator or the CPI.

2.5.2.a Measuring Inflation with GDP deflator

The GDP deflator measures the price level of goods and services of the current year compared to the level of prices of the base year. It is computed as the ratio of Nominal GDP and Real GDP times 100:

$$GDP\ Deflator = \frac{Nominal\ GDP}{Real\ GDP} * 100$$

As explained prior, Nominal GDP reflects both the quantity and prices of goods and services produced in an economy, in contrast with the Real GDP which only reflects the quantity of the goods and services. If the quantity of goods in an economy increases but the prices stay the same, it means the GDP deflator is going to be constant. However, if the quantity of goods remains the same but the overall price level rises, then the GDP deflator is going to increase as it only reflects the changes in the level of prices.

As the inflation rate is considered to be the percentage change in the level of prices, it can be calculated by the GDP deflator by the following formula:

$$Inflation\ rate\ in\ \ t+1 = \frac{GDP\ deflator\ in\ \ t+1 - GDP\ deflator\ in\ t}{GDP\ deflator\ in\ \ t} * 100$$

Where t represents the year.

2.5.2.b Measuring Inflation with CPI

Another way to measure the inflation rate is through CPI. The inflation rate is calculated as the percentage change in the CPI over two consecutive years. It can be computed by the following formula:

$$Inflation\ rate\ in\ year\ t = \frac{CPI\ in\ year\ t + CPI\ in\ year\ t-1}{CPI\ in\ year\ t-1} * 100$$

2.5.2.c CPI vs GDP deflator

Both CPI and GDP deflator are used to determine how fast the level of prices in the economy rises. Usually both measures follow the same pattern, but not always. As mentioned above, CPI and GDP deflator are calculated by different parameters that affect the economy. Sometimes, this may lead these two variables to tell a different story about the inflation rate and create divergence. One main difference that potentially causes the above situation is the fact that GDP deflator reflects the price changes of domestically produced goods and services, whereas CPI reflects the price changes in goods and services acquired by a typical consumer. Thus, domestic products that are not included in a usual consumer purchase will be reflected only in the GDP deflator. Moreover, the purchase of a non-domestic product which is part of the typical consumer's basket of goods will affect the CPI but not the GDP deflator. Another difference between the two indicators is the way the different prices are weighed during the calculation of overall price levels. CPI compares the prices of a fixed basket of goods and services of the current year with the prices of the base year, whereas GDP deflator compares the prices of goods that were produced in the current year with the ones produced in the base year. As the fixed basket of goods and services rarely changes, when the prices are not changing proportionally, the way the two indicators weigh the different prices varies, and their calculations of the inflation rate differs.

2.5.3 Inflation types

There are three main types of inflation observed in the economy; demand pull inflation, cost push inflation and built- in inflation.

2.5.3.a Demand Pull Inflation

Demand Pull inflation can occur when there is an aggressive increase in the demand for goods in the market. A rise in the money supply of an economy can be a factor causing this phenomenon. When people have more money, they tend to spend more, increasing the demand for goods of which supply may be limited. This gap between demand and supply results in higher prices.

2.5.3.b Cost Push Inflation

Cost Push inflation is a result of the rise in the prices of raw materials businesses use to produce their products. If the cost of production increases, then the prices of the final products will increase too, for businesses to maintain their profit. This can occur regardless of the demand in the market.

2.5.3.c Built-in Inflation

When the market is facing high inflation, people tend to expect that the same inflation rates will continue in the future. This could lead to a demand for higher wages and salaries from workers to maintain their standard of living. Employers will raise the salaries to avoid a labor shortage but at the same time the prices of goods will again increase as businesses want to keep their profits at the same levels. This phenomenon is called Built – in inflation.

2.5.3.d Inflation Expectations

Inflation expectations are the expected future inflation by the population. It is of great importance, because the expectations of future inflation affect the current consumers' behavior and has an impact on future inflation. People's beliefs towards inflation affect their present investment decisions. For example, if a lower inflation is expected, people may prefer to proceed with an investment in the future and vice versa; if a higher inflation is expected, a current investment or purchase may be wiser. As mentioned above, people's inflation expectations can determine the future inflation behavior. If firms and workers expect lower future inflation, firms raise their prices at a slower pace and workers don't demand an urgent increase in their wages. This will help to keep a balance in the economy, which will lead to lower inflation in the future. Moreover, to keep the economy in a good position and for sustained growth, the objective inflation for the Federal Reserve is 2%. However, it is difficult to calculate the expected inflation, thus, several surveys and economic models are formed to measure it.

2.5.4 Gross Domestic Product (GDP)

As defined in N. Gregory Mankiw, Gross Domestic Product is "The market value of all final goods and services produced within a country in a given period of time". The GDP measures the total income of every person in an economy and the total expenditure of goods and services produced in all markets in the economy. GDP is the sum of consumption, investment, government purchases and net exports. The rise of the total expenditure in the economy can be attributed to two factors: The increase of the production of goods and services or the rise of the selling prices of goods and services in the economy. To determine and calculate these two situations Real GDP and Nominal GDP were constructed.

2.5.4.a Nominal GDP

Nominal GDP is computed as the sum of all quantity of goods and services produced in an economy multiplied by the price of goods and services in the specific year the GDP is calculated. In other words, Nominal GDP represents the value of the economy's production goods and services in contemporary prices. Thus, a possible rise in the nominal GDP can be attributed to an increase in either the quantity of production or prices of goods and services.

2.5.4.b Real GDP

Real GDP removes the effect of price changes from Nominal GDP and measures only the changes in the production of goods and services. Once, the base year is chosen, the prices of this specific base year are used to calculate the real GDP of the following years. Real GDP is computed as the sum of the quantities of goods and services produced in a specific year multiplied by the prices occurred in the chosen base year. By using constant base year prices, the real GDP reflects only the changes happening in the production of goods and services, which is a better indicator of how well an economy is performing and to what extent people's needs are satisfied. The growth of the economy is measured by the percentage change in real GDP between two consecutive periods.

2.6 Five Theories of Yield Curve

2.6.1 Liquidity Preference Theory

Liquidity Preference Theory hypothesizes that investors prefer to invest for a short period of time to stay "liquid", have more immediate access to their funds and avoid the risk of holding a long-term bond for one period, as the bond price volatility and the bond's maturity are highly correlated. Investors will only prefer to invest in a longer-term maturity bond if the longer-term interest rate is higher.

2.6.2 Inflation Expectations Premium

According to the Inflation Expectations Premium, people feel uncertain about the future rates of inflation and require higher interest rates on longer-term investments.

2.6.3 Pure Expectations Theory

The Pure Expectations Theory is based on the belief that expected future rates are exclusively represented by the forward rates. Thus, the market expectations for the future short term interest rates can be anticipated by the entire term structure, however, this theory is not always reliable as different factors affect the behavior of the short term and long-term interest rates. For example, the Federal Reserve tends to adjust the interest rates which leads to an impact on the bond yields. In addition, long-term yields can also be affected by inflation or economic growth expectations.

2.6.4 Preferred Habitat Theory

The Preferred Habitat Theory is a term structure hypothesis which asserts that some bond investors prefer investing in bonds with a specific maturity and are not willing to invest in assets with different maturities. These investors will consider investing in a different maturity bond group only if a risk premium is attached to these bonds.

2.6.5 Market Segmentation Theory

The Market Segmentation Theory addresses that short term and long-term interest rates are not related. It supports that investors who make up the market of short-term interest rates share characteristics and motivations which differ from the investors in the intermediate and long-term market share. Thus, the short-term interest rates cannot be used to predict long term interest rates or the other way around, as bonds with different maturities should be viewed as items in different markets for debt securities.

2.7 Money Supply

The money supply is the total amount of money in circulation in a country's economy. It consists of both cash and deposits which can be used for payments or as short-term investments. Money supply is a key factor in the macroeconomic performance of a country as it seems to be highly correlated with inflation and price levels. Central bank's monetary regulations determine the money supply in the market with the intent to maintain the good performance of the economy. In the United States, the money supply is determined by the monetary policy of the Federal Reserve. An increase in the money supply could cause a decrease in the interest rates. Such an increase in the money supply would also increase the amount of money in the public and therefore their spending. This will also result in stimulation in production and growth of the demand for labor. If the money supply decreases or its growth rate declines the opposite will be observed.

2.7.1 Measures of Money Supply

The different types of money can be measured according to the size and type of account the instrument is kept. Usually, the money is classified between the Monetary Base, M1 and M2. The Monetary Base is the sum of currency and coins in circulation and central bank reserves. M1 consists of a Monetary Base in addition to the transaction deposits at depository institutions. M2 covers M1 in addition to saving deposits, small-denomination time deposits and money market shares.

2.7.2 Money Supply controlled by Federal Reserve

The Federal Reserve is responsible for controlling the money supply in the economy. The Fed controls the money supply indirectly with actions that will work through the banking system which is responsible for creating money. The Fed controls the money supply by influencing either the quantity of reserves, changing the reserve ratio or changing the discount rate.

The quantity of reserves is controlled by the Fed by buying or selling bonds in the open market or by making loans to banks. By buying bonds from the public in the nation's bond markets, the Fed increases the reserves in banks, thus the amount of money in the economy. Moreover, banks can borrow from the Fed when they are in need of reserves on hand, make new loans, satisfy bank regulations etc. When banks borrow from the Fed they usually pay an interest rate on the loan called the discount rate. With this action, banks increase their reserves, something that allows the banking system to create more money. The Fed can control the money supply by increasing or decreasing this discount rate. A higher rate would discourage banks from borrowing from the Federal Reserve and therefore the number of reserves in the banking system will decrease.

Another way the Fed can control the money supply is through the reserve ratio and the money multiplier. This can be achieved by adjusting the quantity of reserves banks should hold or by changing the interest rate the Fed pays banks for their reserves. Reserve requirements are the minimum amount of reserves a bank should hold against their deposits. If the minimum number of required reserves increases so does the reserve ratio. This will lead to a lower money multiplier and a money supply drop. Since 2008, the Fed started paying interest rates to the banks that hold reserves. By increasing the interest rate on reserves, banks are willing to hold more reserves. This results in the increase of the reserve ratio and the decrease in money supply.

However, the Fed cannot completely control the money supply as there are external factors that are affecting the amount of money in the economy. First, the Fed cannot control the amount of money households are willing to deposit in banks. The more money households choose to deposit, the more the reserves increase in the bank and the more money the banking system can produce. As the decisions regarding household deposits are unpredictable, and out of control, a fall or rise of the money supply may occur without any actions taken by the Fed. A second problem regarding monetary control is the bankers' lending policy which allows banks to create more money only when the deposit money in banks is loaned out. If bankers decide to make fewer loans and hold more reserves, then less money is created, and money supply decreases.

2.7.3 Supply and Demand for Money

Two factors that determine the value of money are the supply and demand for money. Demand for money is the amount of liquid currency people are willing to hold. The level of prices in the economy is one of the most important variables that affect the demand for money. As people use money to purchase goods and services when prices increase, people need more money for their transactions which results in their preference to hold more money in a liquid form instead of investing in assets such as bonds and stocks. With the increase in the level of prices, an increase in the demand for money follows. The money supply, as mentioned above, is controlled and determined by the Fed.

The equilibrium level is the level of prices that balance the supply and demand for money. If the price level is higher than the equilibrium level, people will demand a higher money supply than the Fed can provide. This means that the level of prices should fall to balance the supply and demand for money. However, if the price level is higher than the equilibrium level, the Fed creates more money than people are willing to hold. To maintain the balance between money supply and demand, the prices of goods and services should rise.

2.7.4 Quantitative Theory of Money Supply

The quantitative theory of money supply was originally developed by mathematician Nicolaus Copernicus, but it was popularized later by economists Milton Freidman and Anna Schwartz. According to this theory, the money supply in the economy is proportional to the level of prices of goods and services. A rise in the money supply cannot only affect the level of prices of goods and services but also their supply. As money supply increases the inflation rate increases and leads to the decrease in the value of money. Following is the equation of the quantitative theory of money supply:

$$M * V = P * rGDP$$

$$log(MS * V) = log\ (P * rGDP)$$

$$log(MS) + log(V) = log(P) + log\ (rGDP)$$

Where:

M = Money Supply

V= Velocity of money

P= Price Level

rGDP= real GDP

As the velocity of money and the real GDP are constant, a positive percentage change in the money supply will lead to a positive percentage change in the level of prices (inflation). Thus, inflation is a positive function of the percentage change in money supply.

$$Inflation = \%\Delta(price\ level) = f(\%\Delta(Money\ Supply))$$

2.8 Federal Reserve

2.8.1 Federal Reserve System

A central bank's responsibility is to oversee the banking system and control the quantity of money. The Federal Reserve (Fed) is the US central bank. After a series of bank failures in 1907, the Congress decided that it was necessary to create a central bank that would ensure the health of the banking system of the United States. This resulted in the creation of the Federal Reserve in 1913. The Federal Reserve consists of the Federal Reserve Board of Governors, located in Washington DC, which has up to seven members, and another 12 regional Federal Reserve Banks located all around the country. The Fed has two main responsibilities. Firstly, it monitors the financial condition of each bank, expedites bank transactions and loans to banks who find themselves short of cash, so the economic stability of the banking system is maintained. Secondly, the Federal Reserve is responsible for regulating the money supply in the economy. The set of actions concerning the money supply is called monetary policy and is dictated by the Federal Open Market Committee (FOMC).

2.8.2 Federal Open Market Committee

The Federal Open Market Committee is composed of the members of the Board of Governors and five of twelve regional bank presidents. The decisions of the FOMC determine the increase or decrease of the money supply in the economy. This can happen with the purchase and sale of US government bonds by the Fed. For example, if the FOMC wants to increase the money supply in the economy, the Fed will increase the quantity of money by buying government bonds from the open market. On the other hand, if the FOMC decides to decrease the money supply, the Fed will sell government bonds from its portfolio to the public bond market. These changes in the money supply by the Fed play a decisive role in the economy as they can determine the future inflation, unemployment, and production.

CHAPTER 3: PREVIOUS RESEARCH

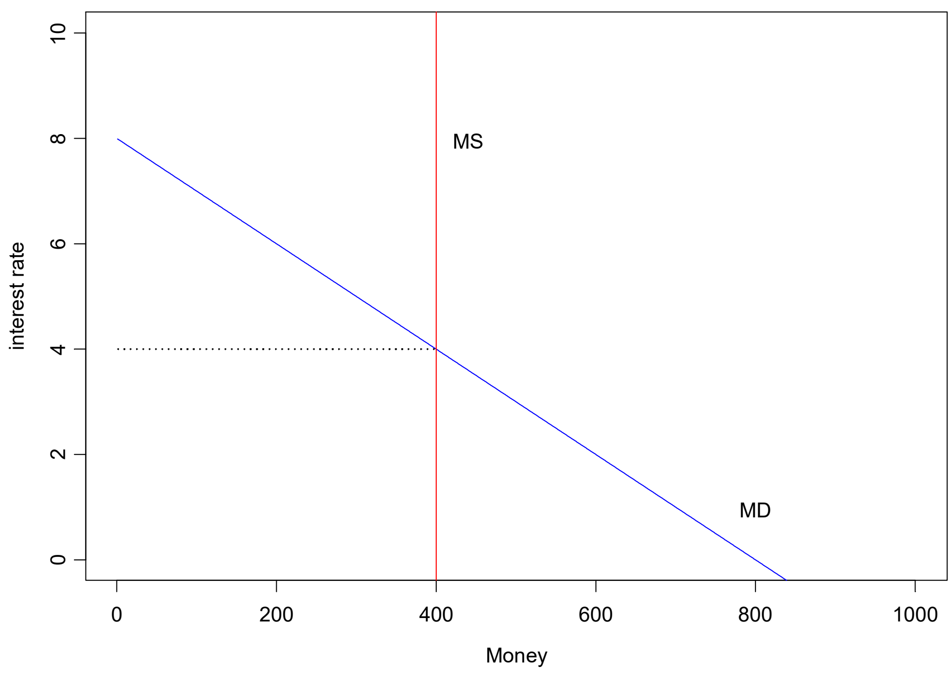
The purpose of this chapter is to describe the relationship between money supply, cpi and inflation with the interest rates and how these relationships are connected to Liquidity Preference theory, Fisher Effect Hypothesis, and negative demand for money. Furthermore, previous research studies, empirical evidence and conclusions regarding the interest rates predictions will be presented.

3.1 Money Supply and Interest Rates

The money supply, as addressed before, is the total amount of money in circulation in a country's economy and it is determined by the Federal Reserve Bank (Fed). Demand for money is the amount of liquid currency people are willing to hold and it is determined by interest rates, economy's population, and income among others. Interest rates ensure that the money supply and demand for money are equal. This equilibrium level is the level of prices where demand and supply of money are equivalent.

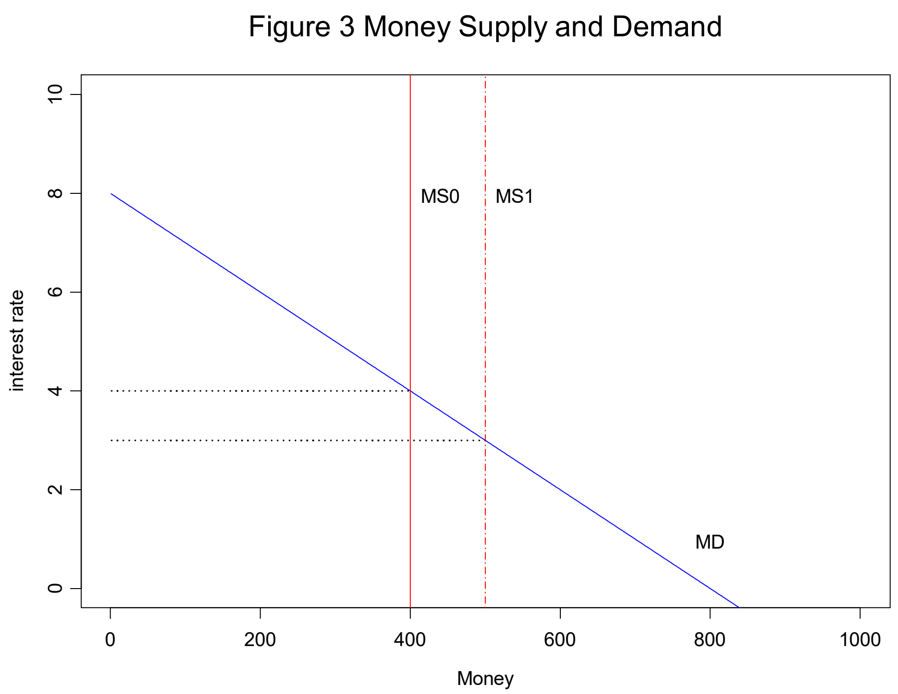
Figure 2 displays the relationship between money supply (red line) and demand (blue line) with the interest rates. The level where the money supply line and demand for money line intersect represents the equilibrium level.

Figure 2 Money Supply and Demand



When the Fed decides to increase the money supply in the economy, the vertical line that represents the money supply will move to the right. In order to keep the balance between money supply and demand, the equivalent level of prices should adjust and thus, interest rates will decrease. On the other hand, if the Fed reduces the money supply, the money supply line will move to the left and the new equivalent level of prices will rise. This will result in higher interest rates.

Figures 3 and 4 display the behavior of the interest rates as the money supply changes.



3.2 Inflation and interest rates

Another important factor that affects the interest rates is inflation. As is theorized by the Fisher effect theory, the nominal interest rate should adjust in a way that it will reflect the changes of the inflation rate and maintain the competitiveness in the market. The Fisher Effect equation reflects the relationship between the nominal (i) and real interest I rate with inflation (π).

$$r = i - \pi$$

As inflation increases, the purchase power of money decreases. Thus, the lender will require a higher interest rate as compensation as the value of money they paid will decrease in the future. Therefore, an increase in inflation will lead to an increase in the interest rates.

3.3 CPI and interest rates

The Consumer Price Index measures the aggregate cost of living of the population in an economy. When the prices of goods and services increase, people will require a higher amount of money for their transactions, something that would lead to a decrease in the real money supply.

Figures 5 and 6 display the behavior of the interest rates as the money supply changes. When the supply for money decreases, the represented line displayed in Figure 5, shifts left and the equilibrium level that balances money supply and demand changes. As shown in Figure 5 the equilibrium level should rise, something that can be accomplished with the increase in the interest rate.

Conversely, when CPI decreases, the money supply increases and the money supply line shifts to the right (Figure 6). In order to keep the money supply and demand equal, the equilibrium interest rate will fall.

Figure 5 Money Supply and Demand

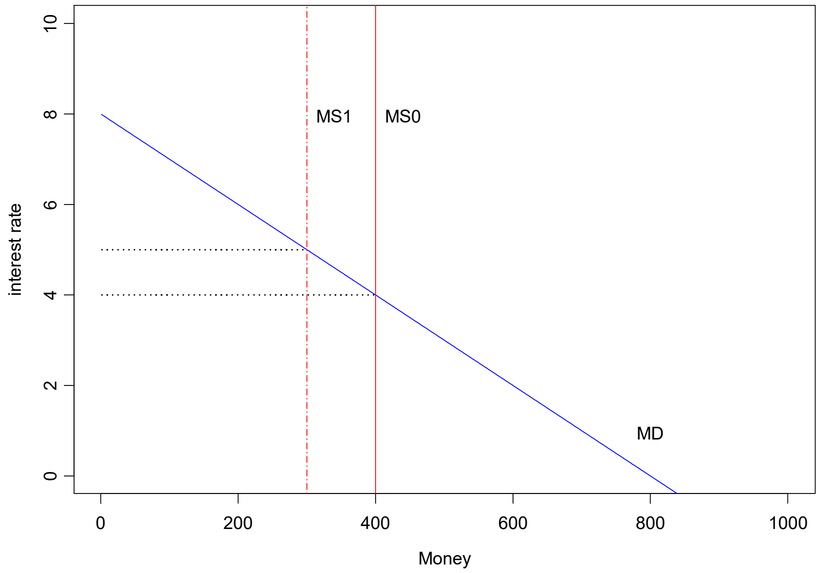
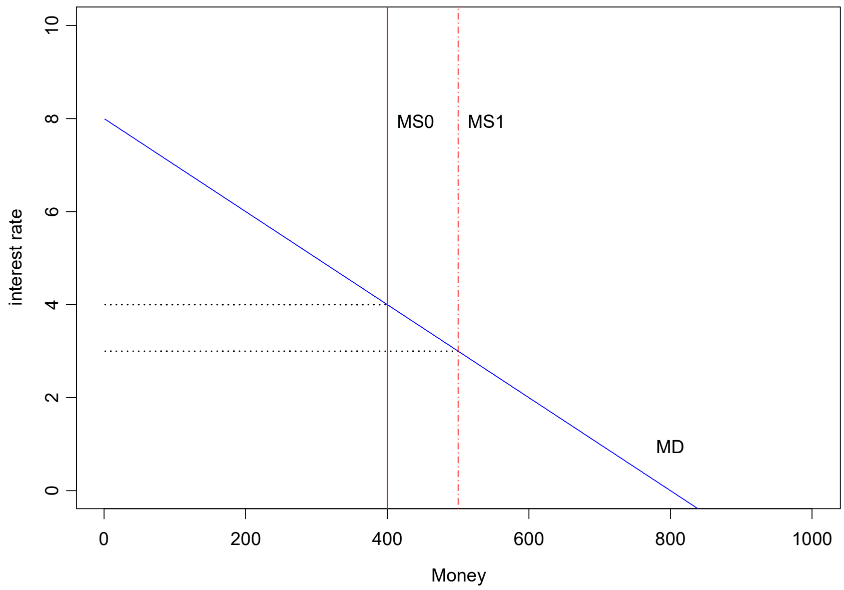


Figure 6 Money Supply and Demand



3.4 Previous Empirical Research

Several empirical studies have been conducted to identify the relationship between interest rates with money supply and inflation.

Fisher (1930) was the first to hypothesize the distinction between nominal and real interest rate. The Fisher Theory Hypothesis supports that there is one to one relationship between the nominal interest rate and inflation because the nominal interest rate is the sum of a constant real interest rate and the expected rate of inflation It also argues that when the money supply affects the inflation rate, it will also affect the inflation expectations.

Crowder and Hoffman (1996) examined the long-run relationship of short-term nominal interest rates and inflation by using time series quarterly data (1952-1991). Their findings indicate a common trend between inflation and three-month treasury bills. They provide evidence which supports that inflation contains information about the future nominal interest rate behavior. Booth and Ciner (2000) investigate the relationship between Eurocurrency nominal interest rates of nine EU countries and the US with their inflation rate. The paper concludes that the interest rate and inflation rate of each country are cointegrated and that a long-run stable relationship exists. This provides support for the long-run Fisher hypothesis. A relationship between interest rates, inflation and money supply within Turkey is examined by the Kaplan and Gungor (2017). The relationship between these variables was analyzed with Cholesky Decomposition Method of Variance based Autoregression Model (VAR) and the Generalized Decomposition Method of Variance. The results of the two methods differ, however, in both cases it is shown that the changes in interest rates can be attributed to the changes in inflation and money supply.

Carr and Smith (1972) examined the theoretical effect of money supply and inflation on interest rates and developed a system of least square equations which uses inflation expectations and the money supply to estimate three dependent variables: the average interest yield on long-term Government of Canada Bonds, the average yield on zero-to-three-year Government Canada Bonds and the 90-day Government of Canada Treasury Bill yield. The model uses the inflation expectations and inflation expectation lags as independent variables. These two independent variables were calculated by using the Almond Interpolation Technique, the difference between the actual quarterly percentage change in money supply between two consecutive periods (t-1, t) and the expected quarterly percentage change in money supply where expectations are formed at time t-1. The results of this paper find a strong negative relationship between the three interest rates and money supply and however a positive relationship between the three interest rates and inflation. Additionally, shorter-term interest rates were identified to be more sensitive to money supply and inflation changes compared to the longer-term ones, with the immediate past inflation expectations having a higher effect on the three interest rates.

Urbanovsky (2016) uses vector autoregression approach to examine the existence of Granger-causality between interest rate, level of money prices and money supply in the Czech Republic. His findings indicate that there is no Granger-causality between money supply change and interest rates changes but the level of prices was identified to Granger-cause interest rates. Moreover, the VAR approach seems unable to predict the magnitude of variable's changes in the study, but successfully predicted the direction of change in interest rates.

Taylor (1992) utilizes weekly data on interest rates of UK short- and long-term government instruments to examine and estimate alternative models for the term structure of interest rates. In this paper the effectiveness of three types of models was investigated: Expectation model, Risk Premium model, and Market Segmentation model. Although, Expectation model and Risk Premium model seem to be unsuccessful in estimating the term structure of interest rates, the empirical formulation of the Market Segmentation approach was more promising because the UK government fixed interest debt outstanding at the relevant maturity, played a significant role in determining the interest rate of return.

CHAPTER 4: DATASET AND PRELIMINARY DATA ANALYSIS

4.1 Dataset

For this research, 412 observations of monthly data were taken from the Federal Reserve Bank of St. Louis Economic Database. The data set runs from 03/1977 to 07/2022 and consists of the Consumer Price Index, the inflation expectations rate, nine Treasury Debt Instrument interest rates and four measures of money supply as follows:

Table 1 Dataset Variables

Names	***Variables***
Three-month Treasury Bill	tb3
Six-month Treasury Bill	tb6
Twelve-month Treasury Bill	irb1
Two-year Treasury Note	irb2
Five-year Treasury Note	irb5
Ten-year Treasury Note	irb10
Twenty-year Treasury Bond	irb20
Thirty-year Treasury Bond	irb30
Adjusted Monetary Base	AMB
M1	M1
M2	M2
M3	M3
Consumer Price Index	cpi
Inflation Expectations	infla24

The percentage change of CPI in the last 24 months was used as a proxy for inflation expectations.

Table 2 displays the dataset used for the research.

Table 2 Dataset

date	*tb3*	*tb6*	*irb1*	*irb2*	*irb5*	*irb7*	*irb10*	*irb20*	*irb30*	*amb*	*m1*	*m2*	*m3*	*ur*	*capu*	*cpi*	*infla24*
1977.03	4.54	4.78	5.40	6.01	6.94	7.21	7.42	7.72	7.79	0.12	0.18	0.24	0.24	7.40	82.81	59.60	0.13
1977.04	4.69	4.92	5.60	6.14	6.90	7.25	7.45	7.72	7.80	0.12	0.18	0.25	0.24	7.20	83.40	60.00	0.13
1977.05	5.03	5.23	5.86	6.18	6.87	7.16	7.38	7.68	7.74	0.12	0.18	0.25	0.25	7.00	83.83	60.20	0.13
1977.06	4.98	5.19	5.72	6.08	6.70	7.00	7.20	7.57	7.58	0.12	0.18	0.25	0.25	7.20	84.24	60.50	0.13
1977.07	5.40	5.70	6.22	6.50	7.03	7.28	7.42	7.68	7.72	0.12	0.18	0.25	0.25	6.90	84.12	60.80	0.13
1977.08	5.56	5.86	6.36	6.59	6.94	7.11	7.28	7.53	7.60	0.12	0.19	0.25	0.25	7.00	83.97	61.10	0.13
.																	
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2022.02	0.37	0.68	1.01	1.44	1.71	1.81	1.83	2.25	2.17	5.98	11.93	4.45	4.42	3.80	79.41	284.18	0.10
2022.03	0.51	1.02	1.63	2.28	2.42	2.40	2.32	2.59	2.44	6.08	11.95	4.46	4.43	3.60	79.82	287.71	0.11
2022.04	0.83	1.37	2.10	2.70	2.92	2.94	2.89	3.14	2.96	5.83	11.90	4.44	4.41	3.60	80.23	288.66	0.13
2022.05	1.13	1.60	2.08	2.53	2.81	2.87	2.85	3.28	3.07	5.54	11.91	4.44	4.42	3.60	80.04	291.47	0.14
2022.06	1.66	2.44	2.80	2.92	3.01	3.04	2.98	3.38	3.14	5.46	11.87	4.43	4.42	3.60	79.84	295.33	0.15
2022.07	2.34	2.81	2.98	2.89	2.70	2.70	2.67	3.20	3.00	5.49	11.85	4.44	4.42	3.50	80.30	295.27	0.14

Graphical techniques (Histograms, Time-series plots, Scatterplots), descriptive statistics, correlation, as well as multiple linear and nonlinear regression analysis were executed. S+ was the statistical and computing program was used to analyze the data.

4.2 Preliminary Data Analysis

4.2.1 Descriptive Statistics

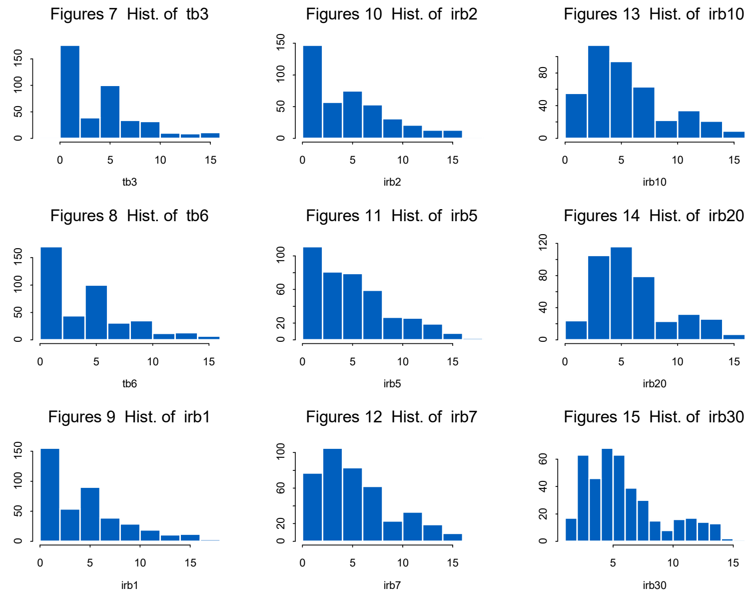
Table 3 displays the Descriptive Statistics of the data. Notice that the mean and median of the debt instrument's ytm increases monotonically with maturity and the standard deviation for all ytm ranges between 3.21 and 4.06. All variables, except M1 and inflation expectations are symmetric and mesokurtic as their skewness and kurtosis are generally between -1 to 1. M1 and inflation expectations skew to the right (skewness equal 3.35 and 1.95 respectively) and are both leptokurtic (kurtosis is 10.09 and 3.31 respectively).

Table 3 Descriptive Statistics

	min	*q1*	*mean*	*med*	*q3*	*max*	*stdev*	*skew*	*kurt*	*n*
tb3	-0.01	0.31	3.92	3.45	5.70	15.52	3.76	0.95	0.35	412
tb6	0.03	0.46	4.04	3.55	5.90	15.69	3.76	0.91	0.18	412
irb1	0.05	0.65	4.40	3.75	6.23	16.97	4.06	0.93	0.21	412
irb2	0.11	0.99	4.65	4.12	6.59	16.73	4.03	0.87	-0.02	412
irb5	0.21	1.90	5.11	4.45	6.95	16.27	3.79	0.89	-0.03	412
irb7	0.39	2.30	5.36	4.55	7.17	16.05	3.66	0.90	-0.02	412
irb10	0.55	2.67	5.54	4.64	7.34	15.84	3.53	0.92	0.01	412
irb20	0.98	3.19	5.94	5.20	7.56	15.78	3.33	0.87	0.00	412
irb30	1.20	3.33	5.95	5.14	7.50	15.19	3.21	0.92	0.04	412
amb	0.12	0.35	1.56	0.71	2.68	6.35	1.68	1.15	0.17	412
m1	0.18	0.56	1.51	0.74	1.41	11.95	2.54	3.35	10.09	412
m2	0.24	0.67	1.47	1.23	2.13	4.46	1.07	1.01	0.35	412
m3	0.24	0.67	1.46	1.22	2.11	4.43	1.07	1.01	0.35	412
cpi	59.60	136.80	178.19	183.65	231.65	295.33	61.70	-0.33	-0.97	412
infla24X	-0.01	0.04	0.07	0.05	0.08	0.27	0.06	1.95	3.31	412

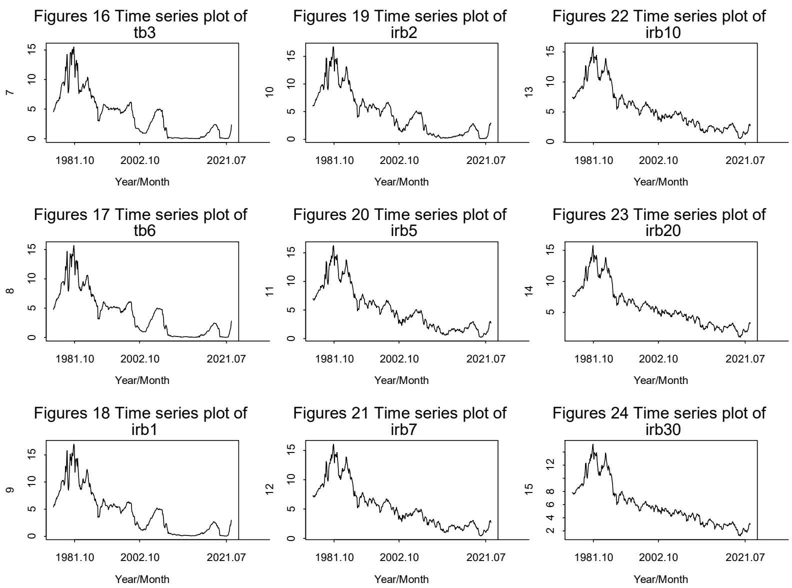
4.2.2 Histograms

Figures 7-15 display the histograms of each debt instrument. The histograms of the interest rates skew to the right but less so as the maturity increases.



4.2.3 Time Series Plots

Figures 16-24 display the time series plots of each debt instrument yield to maturity. All the yield to maturity time series plots peak in 1980, then gradually decrease over time.



4.2.4 Scatterplots

Figures 25-33 display the scatterplots between each debt instrument yield to maturity and the inflation expectations. All yields to maturity have a positive relationship with inflation expectations but nonlinear as they, to a low degree, resemble an S-function.

Figure 25 Plot of tb3 vs inflation

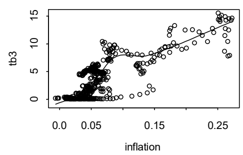


Figure 26 Plot of tb6 vs inflation

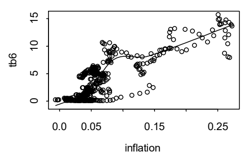


Figure 27 Plot of irb1 vs inflation

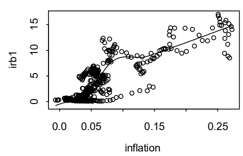


Figure 28 Plot of irb2 vs inflation

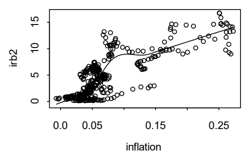


Figure 29 Plot of irb5 vs inflation

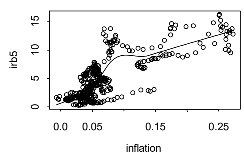


Figure 30 Plot of irb7 vs inflation

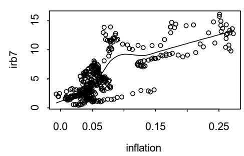


Figure 31 Plot of irb10 vs inflation

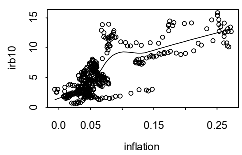


Figure 32 Plot of irb20 vs inflation

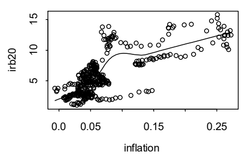
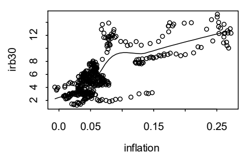


Figure 33 Plot of irb30 vs inflation



Figures 34-42 display the scatterplots between each debt instrument yield to maturity with Adjusted Monetary Base (amb). Interest rates have a negative and asymptotic relationship with amb.

Figure 34 Plot of tb3 vs amb

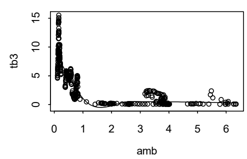


Figure 35 Plot of tb6 vs amb

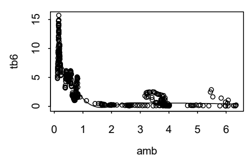


Figure 36 Plot of irb1 vs amb

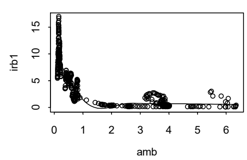


Figure 37 Plot of irb2 vs amb

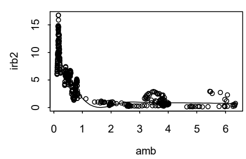


Figure 38 Plot of irb5 vs amb

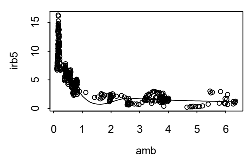


Figure 39 Plot of irb7 vs amb

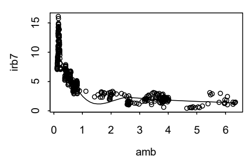


Figure 40 Plot of irb10 vs amb

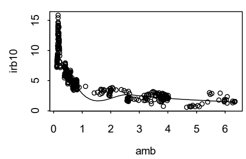


Figure 41 Plot of irb20 vs amb

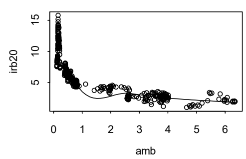
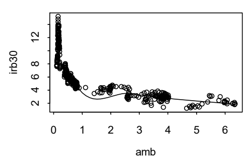


Figure 42 Plot of irb30 vs amb



Figures 43-51 display the scatterplots between each debt instrument yield to maturity with M1. Interest rates have a negative and asymptotic relationship with M1.

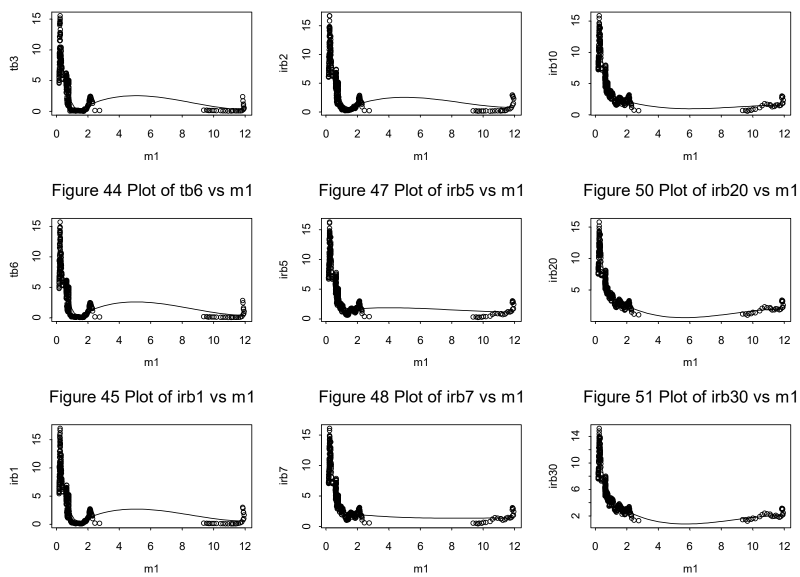


Figure 43 Plot of tb3 vs m1
Figure 46 Plot of irb2 vs m1
Figure 49 Plot of irb10 vs m1
Figure 44 Plot of tb6 vs m1
Figure 47 Plot of irb5 vs m1
Figure 50 Plot of irb20 vs m1
Figure 45 Plot of irb1 vs m1
Figure 48 Plot of irb7 vs m1
Figure 51 Plot of irb30 vs m1

Figures 52-60 display the scatterplots between each debt instrument yield to maturity with M2. Interest rates have a negative and asymptotic relationship with M2.

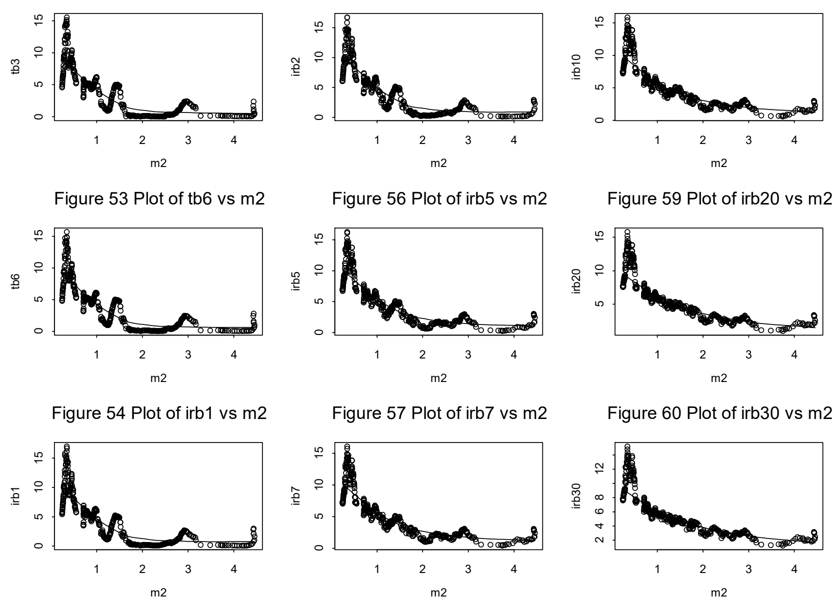


Figure 52 Plot of tb3 vs m2
Figure 55 Plot of irb2 vs m2
Figure 58 Plot of irb10 vs m2
Figure 53 Plot of tb6 vs m2
Figure 56 Plot of irb5 vs m2
Figure 59 Plot of irb20 vs m2
Figure 54 Plot of irb1 vs m2
Figure 57 Plot of irb7 vs m2
Figure 60 Plot of irb30 vs m2

Figures 61-69 display the scatterplot between each debt instrument yield to maturity with the log transformation of amb. The relationship between the two variables remains negative but proves to be more linear than shown in Figures 34-42.

Figure 61 Plot of tb3 vs log(amb)

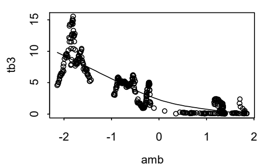


Figure 62 Plot of tb6 vs log(amb)

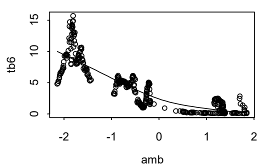


Figure 63 Plot of irb1 vs log(amb)

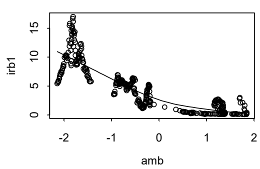


Figure 64 Plot of irb2 vs log(amb)

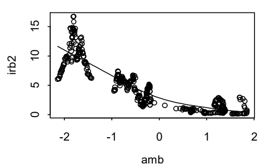


Figure 65 Plot of irb5 vs log(amb)

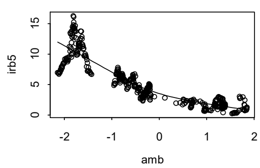


Figure 66 Plot of irb7 vs log(amb)

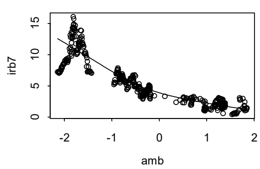


Figure 67 Plot of irb10 vs log(amb)

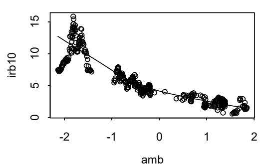


Figure 68 Plot of irb20 vs log(amb)

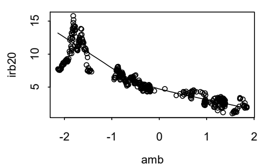
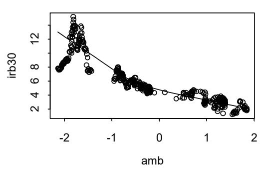


Figure 69 Plot of irb30 vs log(amb)



Figures 70-78 display the scatterplots between each debt instrument yield to maturity and the log transformation of M1. Each ytm remains negative and asymptotic to log(M1).

Figure 70 Plot of tb3 vs log(m1)

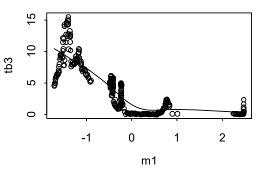


Figure 71 Plot of tb6 vs log(m1)

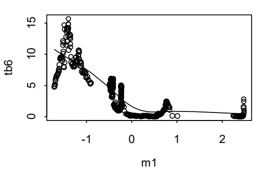


Figure 72 Plot of irb1 vs log(m1)

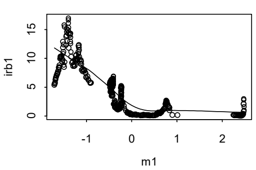


Figure 73 Plot of irb2 vs log(m1)

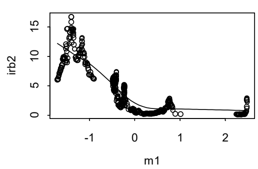


Figure 74 Plot of irb5 vs log(m1)

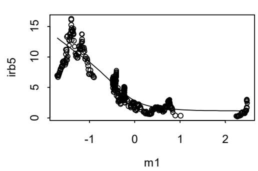


Figure 75 Plot of irb7 vs log(m1)

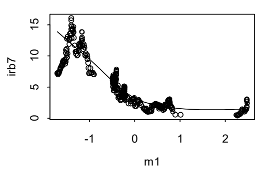


Figure 76 Plot of irb10 vs log(m1)

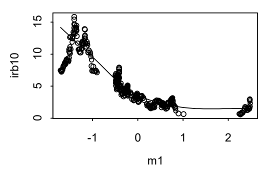


Figure 77 Plot of irb20 vs log(m1)

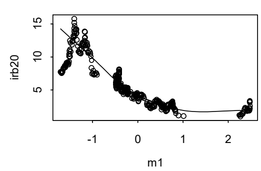
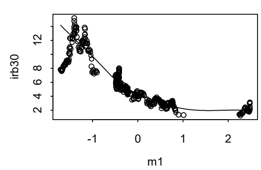


Figure 78 Plot of irb30 vs log(m1)



Figures 79-87 display the scatterplot between each debt instrument yield to maturity with the log transformation of M2. The relationship between the two variables remains negative but it becomes more linear than Figures 52-60.

Figure 79 Plot of tb3 vs log(m2)

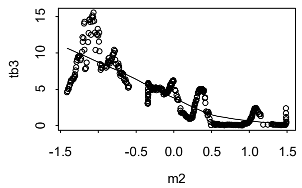


Figure 80 Plot of tb6 vs log(m2)

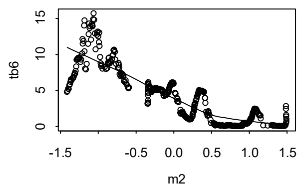


Figure 81 Plot of irb1 vs log(m2)

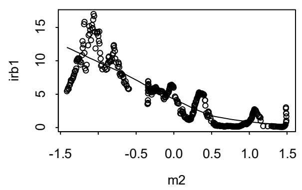


Figure 82 Plot of irb2 vs log(m2)

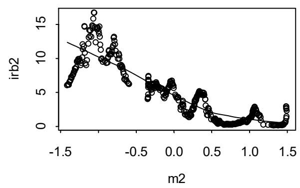


Figure 83 Plot of irb5 vs log(m2)

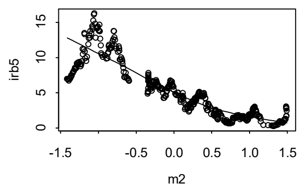


Figure 84 Plot of irb7 vs log(m2)

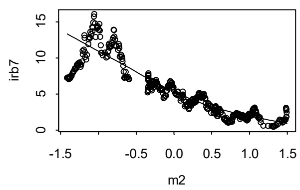


Figure 85 Plot of irb10 vs log(m2)

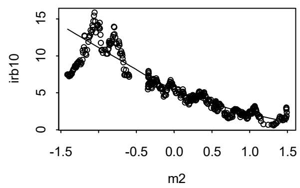


Figure 86 Plot of irb20 vs log(m2)

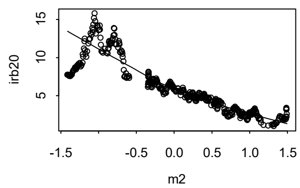
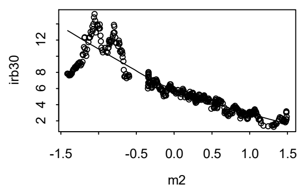


Figure 87 Plot of irb30 vs log(m2)



4.2.5 Correlation

Table 4 displays the Correlation Matrix of the data. As it is shown, all debt instrument interest rates are highly correlated with CPI, inflation expectations and the four measures of money supply. The interest rates have a strong and positive correlation with inflation expectations which decrease with maturity. The correlation between CPI and the ytms is negative and becomes stronger with maturity. Additionally, all measures of money supply have a strong negative relationship with each ytm and their correlation becomes more negative as maturity increases.

Table 4 Correlation Matrix

	AMB	*M1*	*M2*	*M3*	*log(AMB)*	*log(M1)*	*log(M2)*	*log(M3)*	*cpi*	*log(cpi)*	*infla24*	*tb3*	*tb6*	*irb1*	*irb2*	*irb5*	*irb10*	*irb20*	*irb30*
AMB	1	0.760	0.962	0.962	0.913	0.913	0.853	0.853	0.824	0.728	-0.453	-0.682	-0.689	-0.691	-0.706	-0.723	-0.731	-0.749	-0.736
M1	0.760	1	0.772	0.772	0.583	0.807	0.583	0.583	0.543	0.463	-0.228	-0.405	-0.411	-0.413	-0.426	-0.449	-0.467	-0.480	-0.48
M2	0.962	0.772	1	1	0.944	0.966	0.933	0.933	0.910	0.832	-0.532	-0.741	-0.749	-0.754	-0.771	-0.794	-0.809	-0.825	-0.818
M3	0.962	0.772	1	1	0.944	0.966	0.933	0.933	0.910	0.832	-0.532	-0.741	-0.749	-0.754	-0.771	-0.794	-0.809	-0.825	-0.818
log(AMB)	0.913	0.583	0.944	0.944	1	0.932	0.979	0.979	0.973	0.928	-0.667	-0.849	-0.857	-0.862	-0.875	-0.886	-0.89	-0.895	-0.888
log(M1)	0.913	0.807	0.966	0.966	0.932	1	0.941	0.941	0.923	0.879	-0.623	-0.769	-0.777	-0.782	-0.794	-0.813	-0.828	-0.837	-0.836
log(M2)	0.853	0.583	0.933	0.933	0.979	0.941	1	1	0.996	0.972	-0.717	-0.853	-0.861	-0.868	-0.879	-0.891	-0.898	-0.902	-0.900
log(M3)	0.853	0.583	0.933	0.933	0.979	0.941	1	1	0.996	0.972	-0.717	-0.853	-0.861	-0.868	-0.879	-0.891	-0.898	-0.902	-0.900
cpi	0.824	0.543	0.91	0.91	0.973	0.923	0.996	0.996	1	0.983	-0.714	-0.851	-0.859	-0.866	-0.878	-0.891	-0.898	-0.901	-0.900
log(cpi)	0.728	0.463	0.832	0.832	0.928	0.879	0.972	0.972	0.983	1	-0.761	-0.832	-0.84	-0.847	-0.853	-0.861	-0.865	-0.862	-0.866
infla24X	-0.453	-0.228	-0.532	-0.532	-0.667	-0.623	-0.717	-0.717	-0.714	-0.761	1	0.820	0.817	0.812	0.787	0.766	0.757	0.742	0.746
tb3	-0.682	-0.405	-0.741	-0.741	-0.849	-0.769	-0.853	-0.853	-0.851	-0.832	0.82	1	0.999	0.995	0.986	0.966	0.946	0.929	0.925
tb6	-0.689	-0.411	-0.749	-0.749	-0.857	-0.777	-0.861	-0.861	-0.859	-0.84	0.817	0.999	1	0.999	0.992	0.973	0.954	0.937	0.933
irb1	-0.691	-0.413	-0.754	-0.754	-0.862	-0.782	-0.868	-0.868	-0.866	-0.847	0.812	0.995	0.999	1	0.996	0.981	0.964	0.948	0.944
irb2	-0.706	-0.426	-0.771	-0.771	-0.875	-0.794	-0.879	-0.879	-0.878	-0.853	0.787	0.986	0.992	0.996	1	0.993	0.979	0.966	0.962
irb5	-0.723	-0.449	-0.794	-0.794	-0.886	-0.813	-0.891	-0.891	-0.891	-0.861	0.766	0.966	0.973	0.981	0.993	1	0.995	0.988	0.985
irb10	-0.731	-0.467	-0.809	-0.809	-0.89	-0.828	-0.898	-0.898	-0.898	-0.865	0.757	0.946	0.954	0.964	0.979	0.995	1	0.998	0.996
irb20	-0.749	-0.480	-0.825	-0.825	-0.895	-0.837	-0.902	-0.902	-0.901	-0.862	0.742	0.929	0.937	0.948	0.966	0.988	0.998	1	0.999
irb30	-0.736	-0.480	-0.818	-0.818	-0.888	-0.836	-0.900	-0.900	-0.900	-0.866	0.746	0.925	0.933	0.944	0.962	0.985	0.996	0.999	1

CHAPTER 5: ANALYSIS

5.1 Methodology

This research chooses one of five mathematical specifications and one of three measures of money supply that will best model the US Government Yield Curve levels and dynamics. Nine Regression models will be created to estimate each of the nine yields to maturity. For the purpose of this research the underlying data is assumed to be cointegrated.

Eqn. 1 displays the functional specification of the model. It is hypothesized that as inflation or cpi increases, each ytm will increase and as money supply increases each ytm will decrease.

$$ytm_i = f(\overset{-}{\text{money supply}}, \overset{+}{\text{cpi}}, \overset{+}{\text{infla}}) \quad (1)$$

where:

ytm_i = tb3, tb6, us1, us2, us5, us7, us10, us20, us30

infla = % change in consumer price index last 12 months

The five mathematical specifications are:

basic: $ytm_i = a + b_{ms} * ms + b_{infla} * infla$ (2)

logMS: $ytm_i = a + b_{ms} * \ln(ms) + b_{infla} * infla + b_{cpi} * cpi$ (3)

NExp: $ytm_i = a + b_{ms} * \exp(c_{ms} * ms) + b_{infla} * infla + b_{cpi} * cpi$ (4)

sLog: $\ln(ytm_i) = a + b_{ms} * ms + b_{cpi} * cpi + b_{infla} * infla$ (5)

dLog: $\ln(ytm) = a + b_{ms} * \ln(ms) + b_{cpi} * \ln(cpi) + b_{infla} * \ln(1 + infla)$ (6)

where:

ytm_i = tb3, tb6, us1, us2, us5, us7, us10, us20, us30

infla = % change in Consumer Price Index last 12 months

ms = Money Supply

cpi= Consumer Price Index

The three measures of money supply are:

- ❖ AMB (Adjusted Monetary Base) = Currency and Coin
- ❖ M1 = AMB + Checking Accounts
- ❖ M2 = M1 + Saving Accounts

5.2 Model Comparison and Evaluation

As it is shown in Table 4 there is a strong correlation between the independent variables used in each model, something that provides evidence of multicollinearity. However, as the main objective of the optimal model is the prediction of the yield curve, multicollinearity is not our main concern. Each element of Tables 5, 6 and 7 show the average R^2, Median and Mean Absolute Error of the nine debt instruments for each model and each money supply, respectively. As shown in red within Table 5 (R^2), the model with the highest predictive power is the non-linear exponential model ($R^2 = 0.87$). The next most predictive models are the double log linear model and the linear model with the log transformation in money supply ($R^2 = 0.85$). The models with highest predictive power were identified using the Adjusted Monetary Base. Additionally, Tables 6 and 7 show the mean and median absolute error of the double log linear model to be significantly lower when compared to the other models. Thus, the double log linear model and the non-linear model using Adjusted Monetary Base as the measure of money supply, were decided to be the most accurate and optimal models to use for the analysis.

Table 5 R^2

	amb	*m1*	*m2*	*m3*
basic	0.84	0.83	0.83	0.83
logMS	**0.85**	0.83	0.83	0.83
NExp	**0.87**	0.84	0.86	0.86
sLog	0.80	0.76	0.79	0.79
dLog	**0.85**	0.75	0.82	0.82

Table 6 Median Absolute Error

	amb	*m1*	*m2*	*m3*
basic	0.71	0.75	0.71	0.71
logMS	0.75	0.77	0.7	0.7
NExp	0.71	0.75	0.81	0.81
sLog	0.22	0.28	0.24	0.24
dLog	**0.16**	0.32	0.18	0.18

Table 7 Mean Absolute Error

	amb	*m1*	*m2*	*m3*
basic	1.10	1.10	1.11	1.11
logMS	0.81	0.83	0.80	0.80
NExp	0.99	1.12	1.04	1.04
sLog	0.42	0.49	0.46	0.46
dLog	**0.36**	0.51	0.41	0.41

Equations 7 and 8 display the population and sample regression equations for the linear model using the double log transformation. Equations 9 and 10 display the population and sample regression equation for the non-linear model.

$$\ln(ytm_i) = \alpha + \beta_{amb} * \ln(amb) + \beta_{cpi} * \ln(cpi) + \beta_{infla} * \ln(1 + infla) + \varepsilon \quad (7)$$

$$\ln(ytm_i) = a + b_{amb} * \ln(amb) + b_{cpi} * \ln(cpi) + b_{infla} * \ln(1 + infla) + e \quad (8)$$

$$ytm_i = \alpha + \beta_{ms} * \exp(\gamma_{ms} * ms) + \beta_{infla} * infla + \beta_{cpi} * cpi + \varepsilon \quad (9)$$

$$ytm_i = a + b_{ms} * \exp(c_{ms} * ms) + b_{infla} * infla + b_{cpi} * cpi + e \quad (10)$$

5.2.1 Regression Results of Linear Model & Double Log Transformation

Table 8 displays the regression results of the linear model with the double log transformation and using Adjusted Monetary Base as the measure for money supply. Nine regression models were created to compute and predict each interest rate individually. All nine models have a significantly high R^2 which increases with maturity. Importantly, all t-statistics displayed in the last four columns are very high, indicating that all coefficients of the independent variables, Money Supply, inflation, and CPI are statistically significant at 0.01 level, for all models predicting each interest rate. Furthermore, the coefficients for money supply are all negative. This provides evidence that supports the negative demand for money. The coefficients become less negative as the maturity of the interest rate increases. The coefficients for inflation are all positive, which supports the Fisher Equation Hypothesis. The coefficients become less positive with maturity indicating that short-term rates become more sensitive to inflation than long term interest rates are. The coefficients for CPI are all positive and significant. This provides evidence that the negative demand for money hypothesis is a function of the real money supply and not the nominal money supply.

Table 8 Regression Results Linear Double Log Model/ AMB

Instrument	*n*	*Actual Values Jul-22*	*Pred Values Jul-22*	*Pred Values Dec-22*	*rpct*	R^2	*Zero Infla*	*int*	b_{MS}	b_{cpi}	b_{Inlfa}	t_{int}	t_{MS}	t_{cpi}	t_{Inlfa}	*Pred Values 1980*
tb3	413	2.34	0.21	0.24	0.29	0.74	0.08	-19.32	-2.33	3.65	14.17	-9.73	-20.55	9.75	4.92	14.99
tb6	413	2.81	0.34	0.37	0.27	0.77	0.14	-15.65	-1.97	3.00	12.99	-9.77	-21.56	9.92	5.59	14.83
irb1	413	2.98	0.47	0.51	0.23	0.79	0.22	-12.87	-1.73	2.51	11.75	-9.16	-21.62	9.48	5.77	15.57
irb2	413	2.89	0.72	0.77	0.18	0.81	0.39	-8.98	-1.38	1.83	9.19	-8.09	-21.78	8.72	5.71	14.58
irb5	413	2.70	1.39	1.43	0.11	0.85	0.94	-4.02	-0.90	0.96	5.83	-5.77	-22.64	7.35	5.77	13.07
irb7	413	2.70	1.73	1.77	0.11	0.87	1.27	-2.42	-0.74	0.69	4.63	-4.39	-23.61	6.63	5.79	12.57
irb10	413	2.67	2.02	2.06	0.09	0.88	1.54	-1.54	-0.65	0.54	4.03	-3.32	-24.49	6.18	6.00	12.31
irb20	413	3.20	2.52	2.56	0.08	0.90	2.03	-0.89	-0.55	0.45	3.27	-2.41	-26.23	6.39	6.09	11.99
irb30	413	3.00	2.64	2.68	0.08	0.89	2.22	0.12	-0.47	0.26	2.65	0.35	-23.43	3.89	5.17	11.61

Therefore, each ytm is a positive function of inflation and a negative function of the real money supply while the sensitivities decrease for the long-term maturities. Since the model is a logarithmic model, the relationships are geometric and not additive.

5.2.2 Regression Results of Non-Linear Exponential Model

Table 9 displays the regression results of the non-linear exponential model using Adjusted Monetary Base as the measure for money supply. Nine regression models were created to compute and predict each interest rate individually. All nine models have a significantly high R^2 which increases with maturity. Ultimately, all the absolute values for the t-statistics for all independent variables, (i.e., money supply, inflation and cpi) are high. This indicates that all independent variables, for all nine models, are statistically significant at 0.01 level. Moreover, the combination of the money supply coefficient and the exponent shows a statistically significant negative relationship between the money supply and the nine interest rates. Hence, if the money supply increases all interest rates go down. The yield curve would shift or pivot up or down depending on the relative magnitude of the combined coefficient and exponent.

Additionally, all nine models have positive coefficients for cpi and decrease with maturity. It is shown that short-term interest rates are more sensitive to cpi than long-term interest rates. Lastly, all coefficients of inflation, supporting the Fisher Effect Hypothesis, are positive and become significantly small as the maturity increases which implies that inflation has a very small effect on long-term interest rates.

Table 9 displays the regression results of the non-linear exponential model.

Table 9 Regresson Results Non-Linear Exponential Model/ AMB																		
Instrument	*n*	*Actual Values Jul-22*	*Pred Values Jul-22*	*Pred Values Dec-22*	*rpct*	R^2	*Zero Infla*	*int*	b_{MS}	e_{ms}	b_{cpi}	b_{Inlfa}	t_{Int}	t_{MS}	t_{ms}	t_{cpi}	t_{Inlfa}
tb3	413	2.34	4.13	3.96	0.36	0.86	-2.07	-6.97	13.53	-1.70	49.58	0.03	-4.75	9.63	-19.93	12.48	4.32
tb6	413	2.81	4.24	4.10	0.32	0.87	-11.06	-7.31	14.18	-1.72	47.92	0.03	-5.12	10.35	-21.69	12.41	4.76
irb1	413	2.98	4.58	4.46	0.30	0.87	-6.52	-7.94	15.82	-1.79	49.43	0.03	-5.24	10.79	-23.21	12.04	4.95
irb2	413	2.89	4.47	4.47	0.24	0.88	-2.78	-8.34	17.32	-1.83	39.62	0.03	-5.62	12.03	-26.20	9.85	5.59
irb5	413	2.70	4.14	4.11	0.21	0.88	-1.94	-4.98	15.17	-1.85	30.34	0.02	-3.54	11.09	-24.32	7.96	4.11
irb7	413	2.70	3.96	3.89	0.19	0.87	0.82	-2.72	13.47	-1.87	26.79	0.02	-1.96	9.97	-21.97	7.12	2.87
irb10	413	2.67	3.82	3.68	0.16	0.87	0.00	-0.38	11.41	-1.89	25.89	0.01	-0.28	8.62	-19.09	7.03	1.47
irb20	413	3.20	3.82	3.63	0.12	0.86	-1.28	1.97	9.20	-1.76	23.20	0.00	1.50	7.26	-15.42	6.51	0.16
irb30	413	3.00	3.72	3.51	0.11	0.86	-1.97	3.36	8.20	-1.94	21.14	0.00	2.60	6.49	-14.50	6.02	-0.69

5.3 Residual Analysis

5.3.1 Residual Analysis of the Linear Model & Double Log Transformation

Scatterplots of the Residuals

Figures 88-96 display the scatterplots of the residuals vs the fitted values for each debt instrument. All scatterplots are well behaved as the residuals for all debt instruments seem to be distributed randomly around the zero-horizontal line. This suggests that the assumption for linearity is reasonable. Moreover, as the maturity of the debt instruments increases, the residuals form a horizontal band around the 0 line. This suggests that the variances of the error terms are equal. Additionally, no outliers were detected in any of the following graphs.

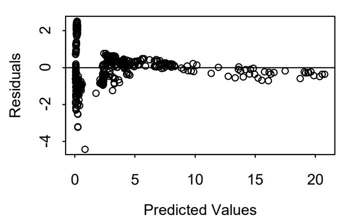


Figure 88 Residuals for tb3

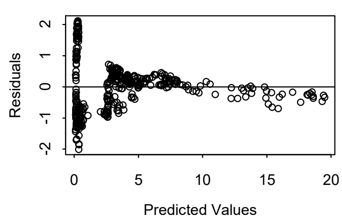


Figure 89 Residuals for tb6

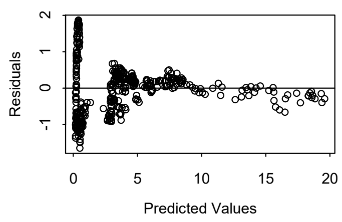


Figure 90 Residuals for irb1

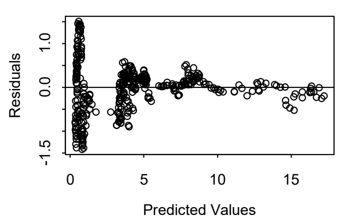


Figure 91 Residuals for irb2

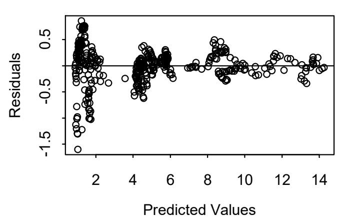


Figure 92 Residuals for irb5

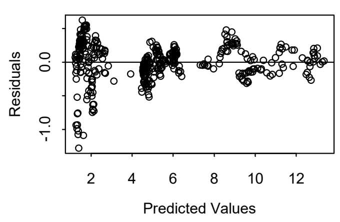


Figure 93 Residuals for irb7

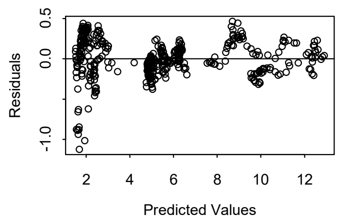


Figure 94 Residuals for irb10

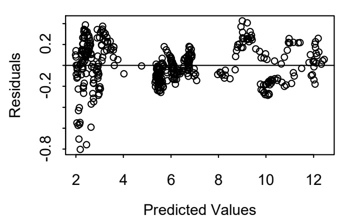


Figure 95 Residuals for irb20

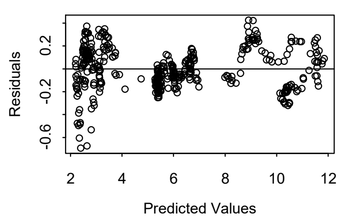


Figure 96 Residuals for irb30

Time series plots of Actual and Predicted values

Figures 97-105 display the time series plots of actual and predicted values for each debt instrument. The pattern of the residuals is evident over time in the following graphs. This is evidence of serial correlation.

Figure 97 Time series plot of Pred & Actual tb3

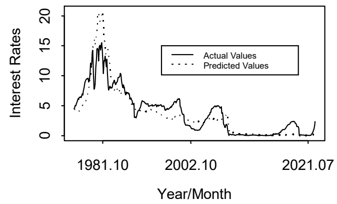


Figure 98 Time series plot of Pred & Actual tb6

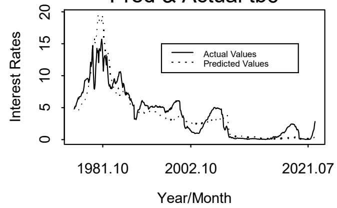


Figure 99 Time series plot of Pred & Actual irb1

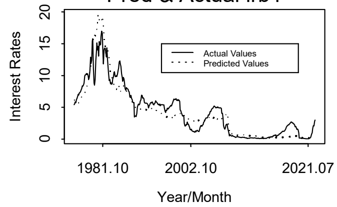


Figure 100 Time series plot of Pred & Actual irb2

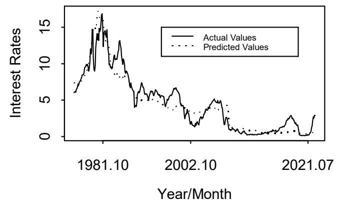


Figure 101 Time series plot of Pred & Actual irb5

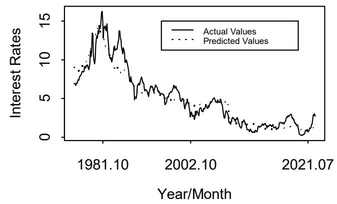


Figure 102 Time series plot of Pred & Actual irb7

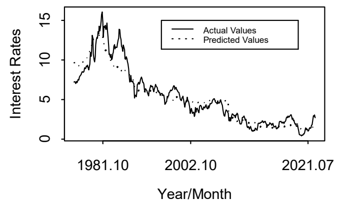


Figure 103 Time series plot of Pred & Actual irb10

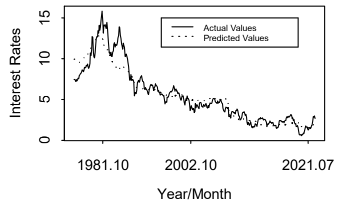


Figure 104 Time series plot of Pred & Actual irb20

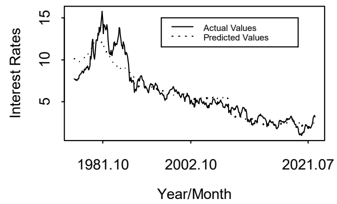
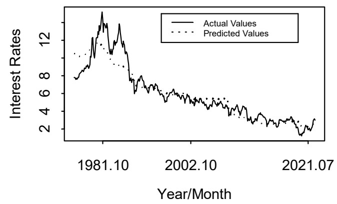


Figure 105 Time series plot of Pred & Actual irb30



Time series plots of Residuals

Figures 106-114 display the time series plots of the residuals for each debt instrument. The serial correlation is even more evident in the following graphs.

Figure 106 Time series plot of Residuals for tb3

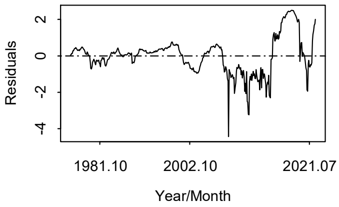


Figure 107 Time series plot of Residuals for tb6

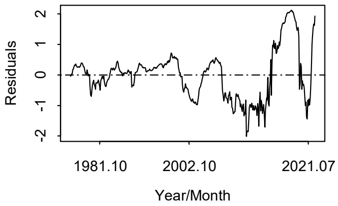


Figure 108 Time series plot of Residuals for irb1

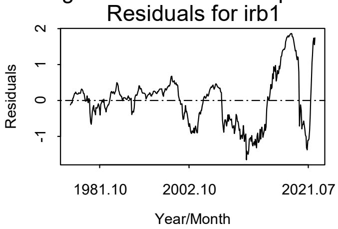


Figure 109 Time series plot of Residuals for irb2

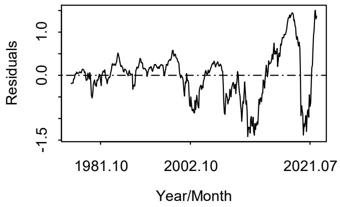


Figure 110 Time series plot of Residuals for irb5

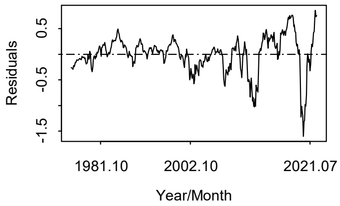


Figure 111 Time series plot of Residuals for irb7

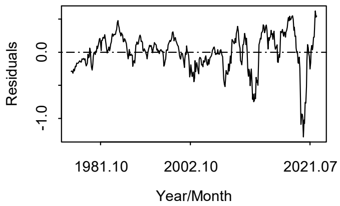


Figure 112 Time series plot of Residuals for irb10

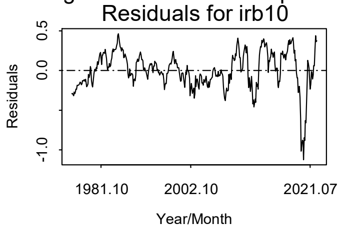


Figure 113 Time series plot of Residuals for irb20

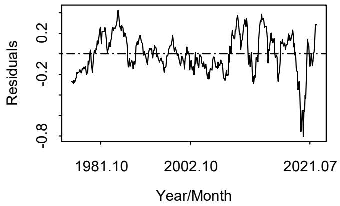
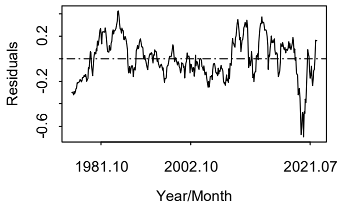


Figure 114 Time series plot of Residuals for irb30



Density Plots of Residuals

Figures 115-123 display the density plots of the Residuals for each debt instrument. All density plots are skewed to the left and become more symmetrical as the instrument maturity increases. The Residuals for all debt instruments seem to follow the Standard Normal Distribution.

Figure 115 Density of Residuals tb3

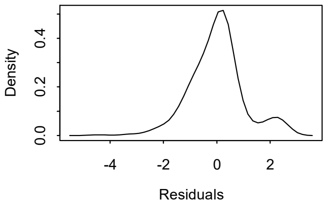


Figure 116 Density of Residuals tb6

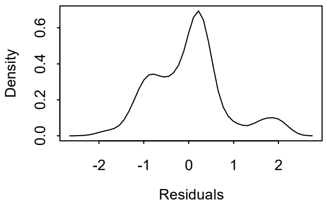


Figure 117 Density of Residuals irb1

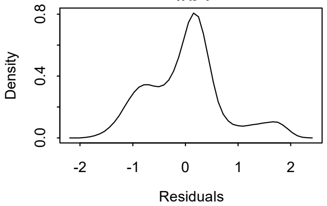


Figure 118 Density of Residuals irb2

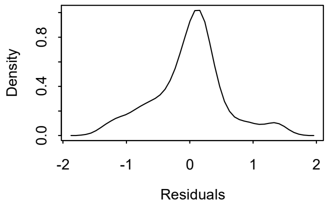


Figure 119 Density of Residuals irb5

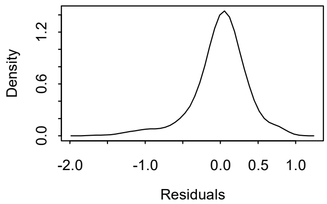


Figure 120 Density of Residuals irb7

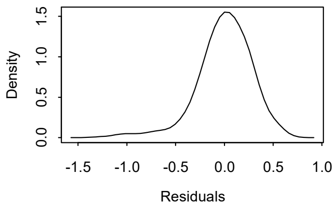


Figure 121 Density of Residuals irb10

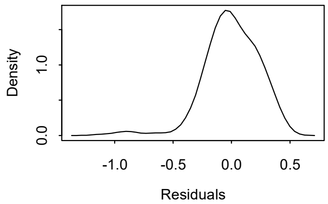


Figure 122 Density of Residuals irb20

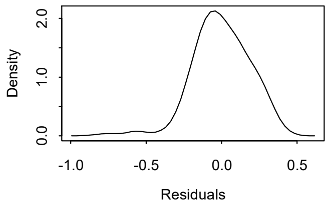
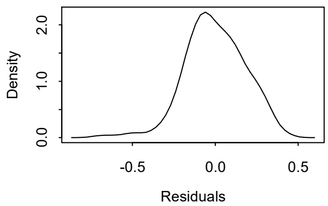


Figure 123 Density of Residuals irb30



Normal QQ-Plots of Residuals

Figures 124-132 display the QQ-Plots between the Residuals and quantiles of Standard Normal. The relationships do not seem to be linear. This indicates that the residuals slightly deviate from a standard Normal Distribution.

Figure 124 Normal QQ plot for tb3

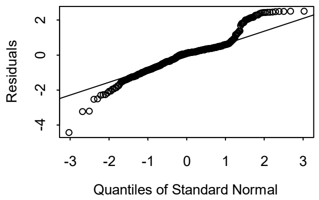


Figure 125 Normal QQ plot for tb6

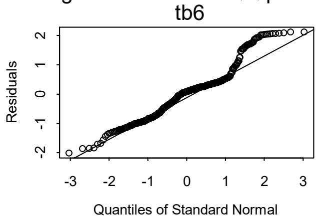


Figure 126 Normal QQ plot for irb1

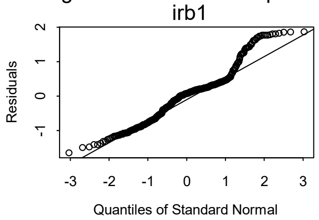


Figure 127 Normal QQ plot for irb2

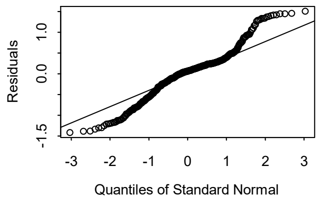


Figure 128 Normal QQ plot for irb5

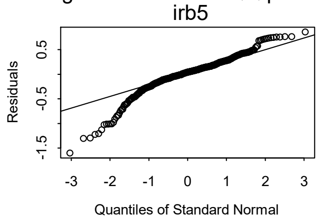


Figure 129 Normal QQ plot for irb7

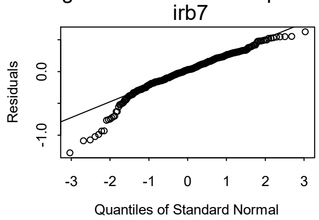


Figure 130 Normal QQ plot for irb10

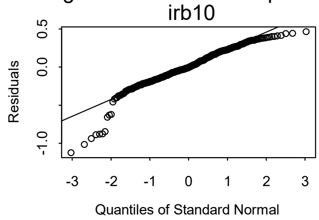


Figure 131 Normal QQ plot for irb20

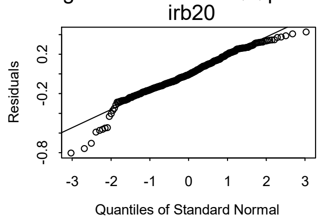
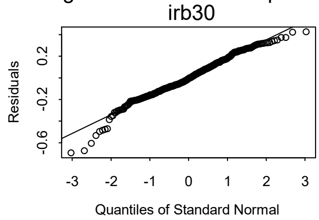


Figure 132 Normal QQ plot for irb30



5.3.2 Residual Analysis of the Non-Linear Exponential Model

Scatterplots of the Residuals

Figures 133-141 display the scatterplots of the Residuals vs the fitted values for each debt instrument by using the non-linear exponential model. All scatterplots are well behaved, as the Residuals for all debt instruments seem to be randomly distributed around the zero-horizontal line. This suggests that the assumption for linearity is reasonable. However, as the maturity of the debt instruments increases, the residuals around the horizontal 0 line spread out more along the x-axis, which suggests that the variances of the error terms are not equal and gives evidence of heteroscedasticity. Additionally, some outliers were detected in all of the following graphs.

Figure 133 Residuals for tb3

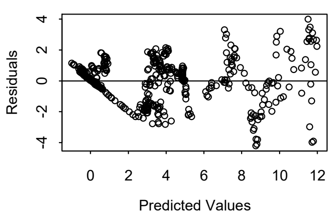


Figure 136 Residuals for irb2

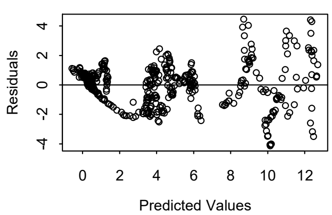


Figure 139 Residuals for irb10

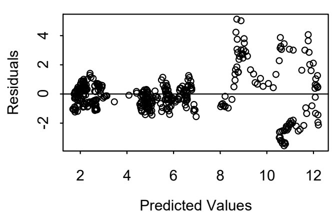


Figure 134 Residuals for tb6

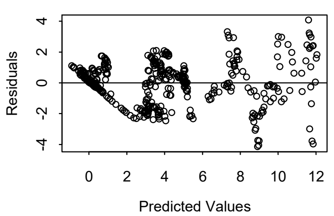


Figure 137 Residuals for irb5

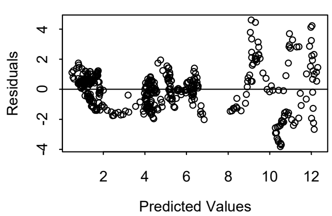


Figure 140 Residuals for irb20

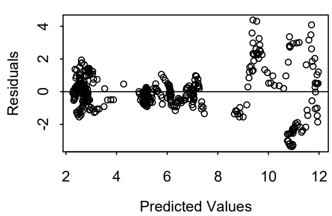


Figure 135 Residuals for irb1

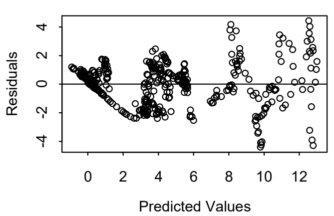


Figure 138 Residuals for irb7

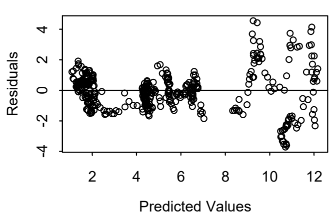
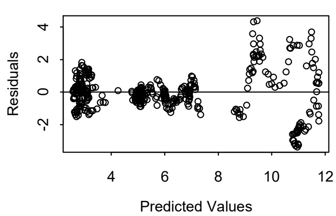


Figure 141 Residuals for irb30



Time Series plots of Actual and Predicted values

Figures 142-150 display the time series plots of actual and predicted values for each debt instrument.

The pattern of the residuals is evident over time in the following graphs. This is evidence of serial correlation.

Figure 142 Time series plot of Pred & Actual tb3

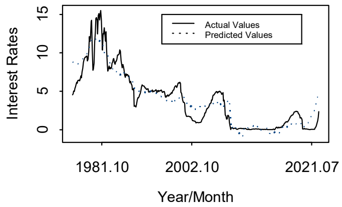


Figure 143 Time series plot of Pred & Actual tb6

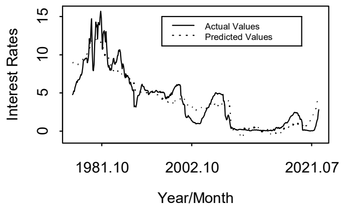


Figure 144 Time series plot of Pred & Actual irb1

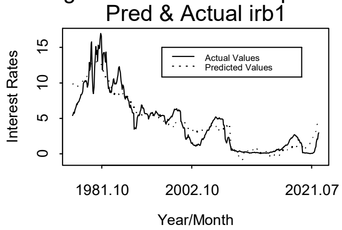


Figure 145 Time series plot of Pred & Actual irb2

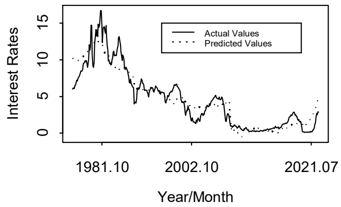


Figure 146 Time series plot of Pred & Actual irb5

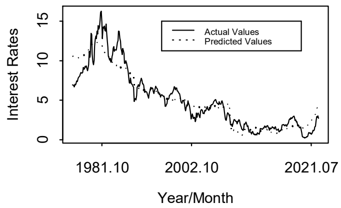


Figure 147 Time series plot of Pred & Actual irb7

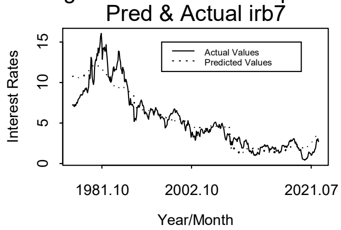


Figure 148 Time series plot of Pred & Actual irb10

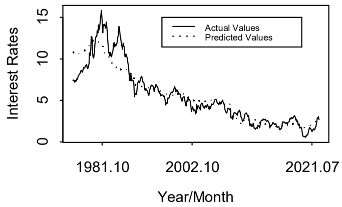


Figure 149 Time series plot of Pred & Actual irb20

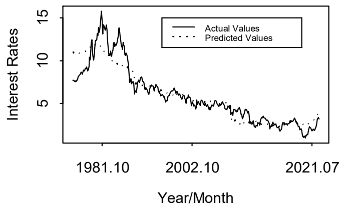
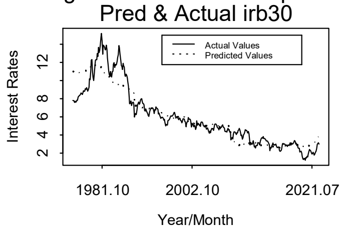


Figure 150 Time series plot of Pred & Actual irb30



Time series plots for the residuals

Figures 151-159 display the time series plots of the residuals for each debt instrument. The serial correlation is even more evident in the following graphs.

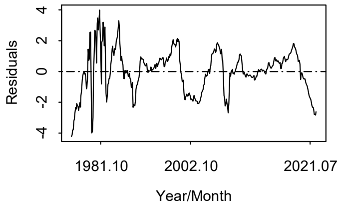


Figure 151 Time series plot of Residuals for tb3

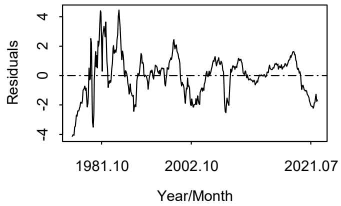


Figure 154 Time series plot of Residuals for irb2

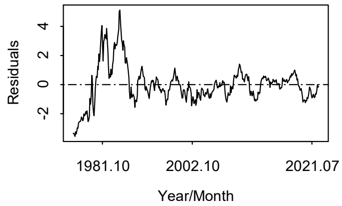


Figure 157 Time series plot of Residuals for irb10

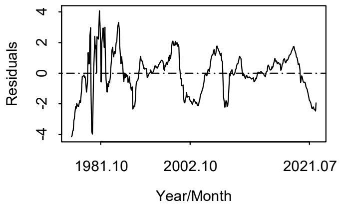


Figure 152 Time series plot of Residuals for tb6

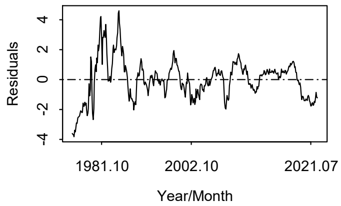


Figure 155 Time series plot of Residuals for irb5

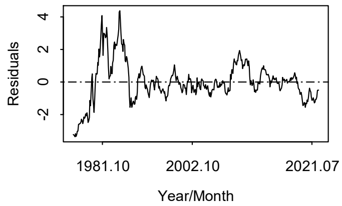


Figure 158 Time series plot of Residuals for irb20

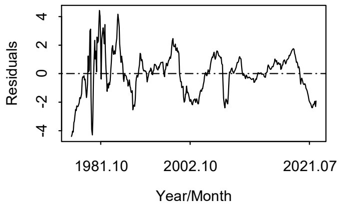


Figure 153 Time series plot of Residuals for irb1

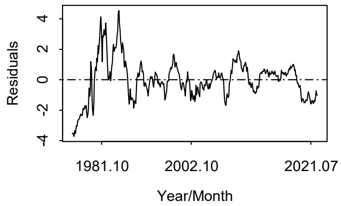


Figure 156 Time series plot of Residuals for irb7

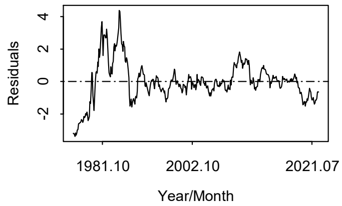


Figure 159 Time series plot of Residuals for irb30

Density plot of residuals

Figures 160-168 display the density plots of the Residuals for each debt instrument. All density plots are skewed to the left and become more symmetrical as the maturity increases. The Residuals for all debt instruments seem to follow the Standard Normal Distribution.

Figure 160 Density of Residuals tb3

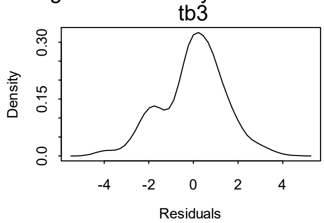


Figure 161 Density of Residuals tb6

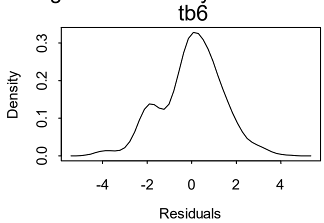


Figure 162 Density of Residuals irb1

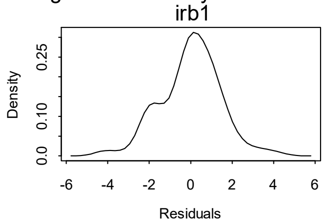


Figure 163 Density of Residuals irb2

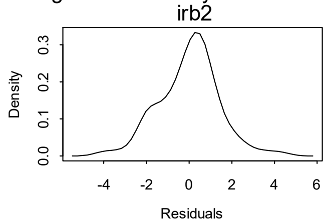


Figure 164 Density of Residuals irb5

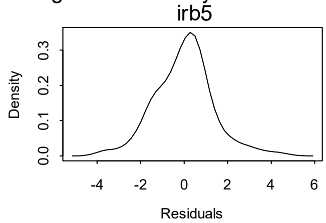


Figure 165 Density of Residuals irb7

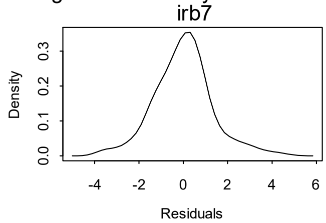


Figure 166 Density of Residuals irb10

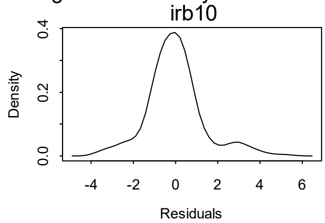


Figure 167 Density of Residuals irb20

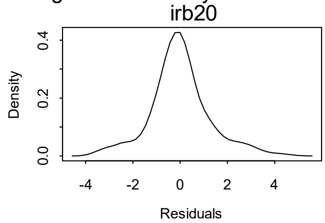
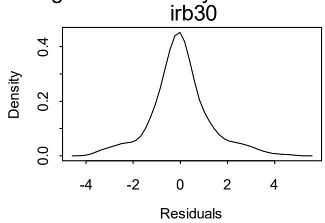


Figure 168 Density of Residuals irb30



QQ-plot of residuals

Figures 169-177 display the QQ-Plots between the Residuals and quantiles of Standard Normal. It seems to be a mostly linear relationship between the quantiles of Standard Normal and the Residuals however, as the maturity of the debt instruments increases, the relationship becomes less linear and transforms to an S-function.

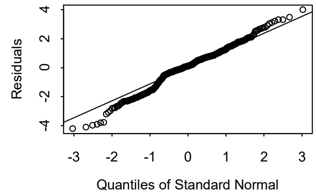


Figure 169 Normal QQ plot for tb3

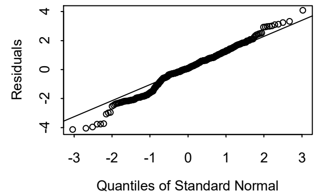


Figure 170 Normal QQ plot for tb6

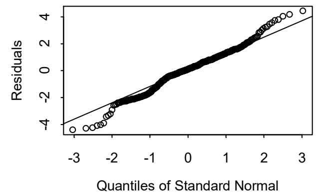


Figure 171 Normal QQ plot for irb1

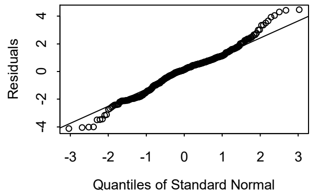


Figure 172 Normal QQ plot for irb2

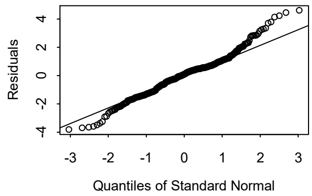


Figure 173 Normal QQ plot for irb5

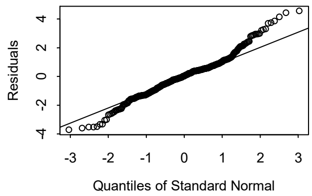


Figure 174 Normal QQ plot for irb7

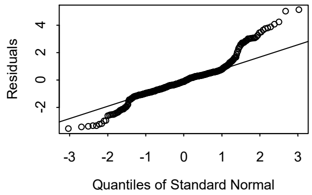


Figure 175 Normal QQ plot for irb10

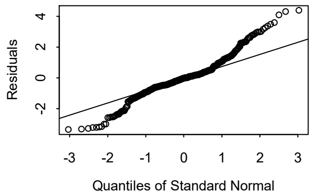


Figure 176 Normal QQ plot for irb20

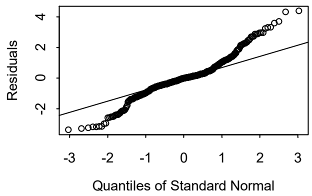


Figure 177 Normal QQ plot for irb30

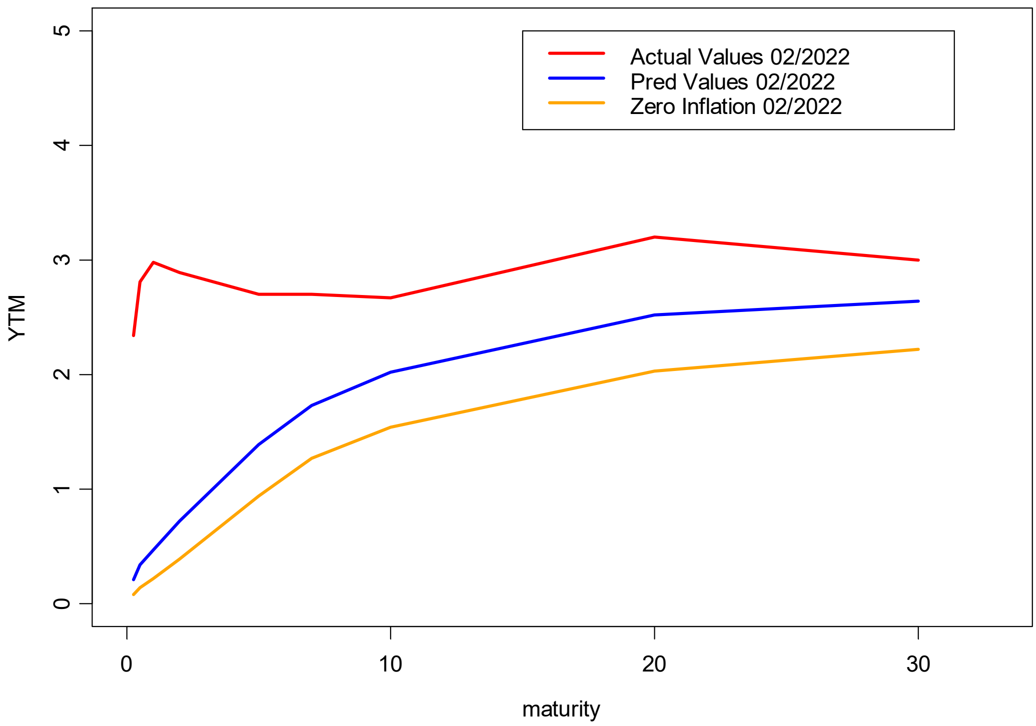
5.4 Selection of the optimal model

The linear model with the double log transformation was chosen as the optimal model to use in predicting the nine interest rates. This model was chosen as it was associated with the minimum median and mean absolute error of all models developed for this research. The following sections display the optimal model’s predicted values compared to actual values for the nine interest rates at different points in time.

5.5 Yield Curve Graphs

Figure 178 displays the Actual Yield Curve on 02/2022 with the red line, the estimated Yield Curve on 02/2022 with the blue line and the estimated "Real" Yield Curve on 02/2022, where inflation is equal to zero with the orange line. The optimal model's estimated interest rates were lower than the actual interest rates. However, it is important to mention that as the maturity of the interest rates increases, the predicted values become more accurate and approach the actual values. Additionally, the zero-inflation real yield curve was estimated and displayed. As expected, all real interest rates were calculated and are much lower compared to the nominal interest rates because, by definition, nominal interest rates are equal to the real interest rates and inflation expectations.

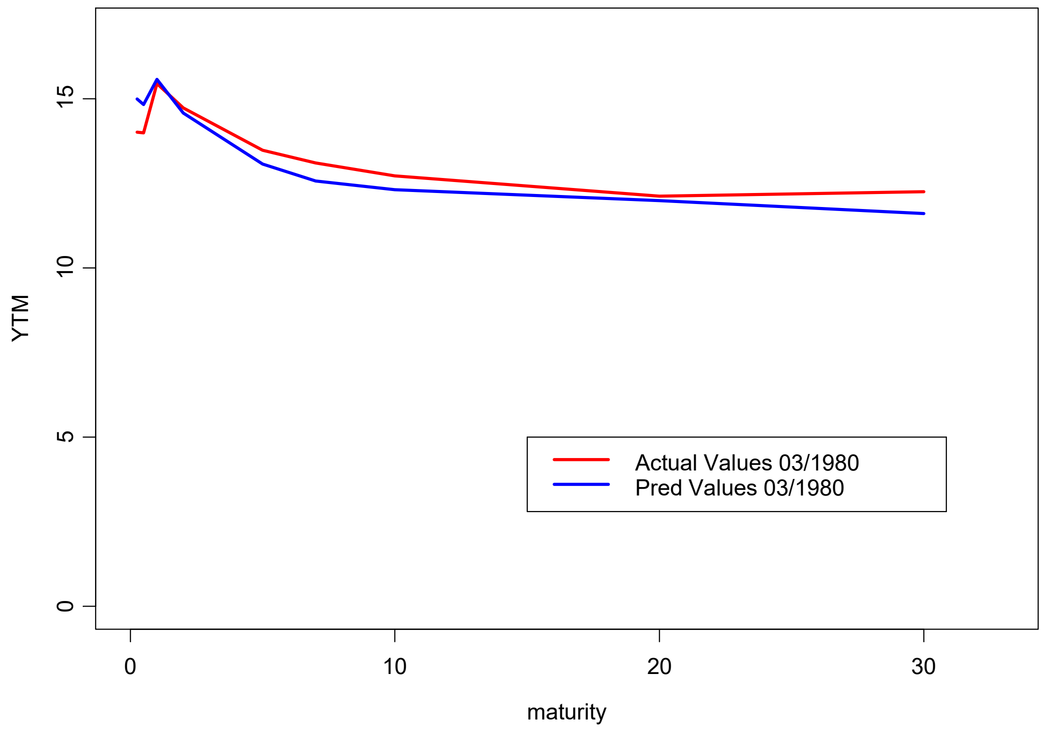
Figure 178 Yield Curves



Predicted values vs Actual values 1980 graph (inverted yield curve).

Figure 179 displays the actual (red) and predicted (blue) yield curves for 03/1980. Even though the inverted yield curve shape was observed only 13 times since 1977, the chosen optimal model was able to identify and predict the inverted shape of the yield curve in 03/1980.

Figure 179 Yield Curves 1980



CHAPTER 6: SUMMARY AND CONCLUSIONS

6.1 Summary

This research uses five mathematical specifications, four measures of money supply and inflation expectations to model nine debt instrument interest rates. The optimal model to predict the nine yields to maturity was identified to be the linear model using the double logarithmic transformation and the Adjusted Monetary Base as the superior measure of the money supply. The optimal model was not only able to successfully predict the nine interest rates but also has the ability to identify the yield curve shape at any specified time period.

The real yield curve, where inflation is equal to zero, was estimated using the chosen optimal model. The values of the real interest rates are all positive and increase with maturity. Short-term real interest rates are predicted to be lower than the long-term real interest rates. This phenomenon supports the Liquidity Preference Theory

Additionally, interest rates were identified to have a positive relationship with inflation expectations. This provides evidence that supports the Fisher Equation Hypothesis. Interest rates and money supply were determined to have a negative or inverted relationship. This supports the negative demand for real money hypothesis. Interest rates and the Consumer Price Index have a positive relationship. This occurs via real money supply effect. As the CPI increases, the real Adjusted Monetary Base decreases, which results in a rise in the interest rates. Ultimately, short-term interest rates were identified to be more sensitive to inflation, money supply and CPI than long-term interest rates.

6.2 Conclusions

The Federal Reserve has two coequal mandates, to maximize employment and stabilize prices. The Fed regulates the financial system by increasing or decreasing the money supply in the economy. A change in the money supply will directly affect the levels of the interest rates and at the same time the level of investment in the market. However, as the Quantitative Theory of Money Supply suggests, a positive percentage change in the money supply will lead to a positive percentage change in the level of prices and therefore inflation. Even though an increase in the money supply will lead to lower interest rates and lower levels of unemployment, it could also result in a substantial increase in inflation. The relationships between money supply, inflation and interest rates identified in this research can assist the Fed in choosing the most optimal monetary policy and ultimately lead to a balanced and healthy economy.

6.3 Further Research

For the purpose of this research, it is assumed that the data is cointegrated. However, as referred to and shown in Chapter 5, all the variables have either an upward or a downward trend. This raises the possibility that the data may not be cointegrated. Further research might use the Dickey-Fuller statistical test to check whether the data is stationary or not. If it is non-stationary, the Johansen statistical test could verify whether the data is cointegrated.

This research used the percentage change of Consumer Price Index over the last twenty-four months as a proxy for inflation expectations. Further research might investigate alternative measures of inflation expectations such as the distributed lagged values of inflation.

Finally, the Federal Funds Rate (FFR) is the interest rate at which depository institutions lend each other reserve balances held overnight by the Federal Reserve. When the Fed raises the FFR through monetary policy, interest rates increase, and the US Dollar becomes stronger. Considering the enormous impact FFR changes have on future short term interest rates and the yield curve, the use of the FFR as an explanatory variable in the model could lead to more accurate estimations of the treasury debt instrument's ytm and increase the model's predictive ability.

APPENDIX

```
kddf<-KATERINADaily
names(kddf)
names(kddf)<-
   c("date","oilb","oilw","irb1","irb10","irb2","irb20","irb30","irb5","irb7","
   tb3","tb6")
kddf<-
   kddf[,c("date","tb3","tb6","irb1","irb2","irb5","irb7","irb10","irb20","irb3
   0","oilb","oilw")]
kddf$date<-fym710(kddf$date)
kddf$mth<-round((kddf$date-round(kddf$date))*100)
kddf$dmth<-round(fdiff(kddf$mth,1))
kddf$syc<-kddf$irb10-kddf$irb2 #slope of the yeild curve
kddf$rowSum<-apply(kddf[,4:10],1,sum,na.rm=T)
kddf<-kddf[kddf$rowSum>1,]
names(kddf)
kddf
kddf<-kddf[!is.na(kddf$irb30),]
kddf$dmth<-flag(kddf$dmth,-1)
kddf
kddf<-kddf[kddf$dmth!=0,]
names(kddf)
kddf
kddf<-kddf[,1:10]
names(kddf)
kddf<-na.omit(kddf)
kddf

# Monthly
kmdf<-KATERINAMonthly
names(kmdf)
names(kmdf)<-
   c("date","aherti","amb","cpi","irff","m1","m2","m3","curcoin","capu","ur","w
   tisplc")
kmdf<-kmdf[,c("date","irff","curcoin","amb","m1","m2","m3",,"ur","capu","cpi")]
kmdf$date<-fym710(kmdf$date)
names(kmdf)
kmdf$curcoin<-kmdf$curcoin/mean(kmdf$curcoin,na.rm=T)
kmdf$amb<-kmdf$amb/mean(kmdf$amb,na.rm=T)
kmdf$m1<-kmdf$m1/mean(kmdf$m1,na.rm=T)
kmdf$m2<-kmdf$m2/mean(kmdf$m2,na.rm=T)
kmdf$m3<-kmdf$m3/mean(kmdf$m3,na.rm=T)
kmdf$infla<-fpctcng(kmdf$cpi,12)   #functions file
kmdf$infla12<-flag(kmdf$infla,12)
kmdf$infla24<-flag(kmdf$infla,24)
kmdf$infla36<-flag(kmdf$infla,36)
kmdf$infla24X<-fpctcng(kmdf$cpi,24)   #functions file
head(kmdf,10); tail(kmdf,10);

#merge
kddf$date; head(kddf); tail(kddf); names(kddf);
kmdf$date; head(kmdf); tail(kmdf); names(kmdf);
kmdf<-kmdf[kmdf$date>1977.02,]
kdf<-merge(kddf,kmdf,by="date",all=T)
qdsdf<-kdf # do not delete this line
kdf<-na.omit(kdf)
names(kdf)
head(round(kdf,2))
tail(round(kdf,2))
```

```
sycdf<-kdf
sycdf$syc<-sycdf$irb10-sycdf$irb2
sycdf<-sycdf[sycdf$syc<0,]
dim(sycdf)
invertYCdf<-sycdf[sycdf$syc==min(sycdf$syc),]
dim(invertYCdf)
invertYCdf


#############################
kdf1<-
   kdf[,c("date","curcoin","amb","m1","m2","m3","tb3","tb6","irb1","irb2","irb5
   ","irb7","irb10","irb20","irb30","cpi","infla24X")]
kdf1<-na.omit(kdf1)

fdesstat(kdf)


#Histograms
names(kdf1)
i<-8
par(mfcol=c(3,3)); for (i in 7:15){hist(kdf1[,i],xlab=
   names(kdf1[i]),main=paste("Fig.",i-2," Hist. of ",names(kdf1)[i]))}
#Time series plots
#par(mfcol=c(3,3)); for (i in 7:15){ts.plot(kdf1[,i],ylab=
   names(kdf1[i]),main=paste("Fig.",i-6+9+1," Time series of
   ",names(kdf1)[i]))}
par(mfcol=c(3,3)); for (i in
   7:15){plot(1:nrow(kdf1),kdf1[,i],xlab="Year/Month",ylab=i,main=paste("Fig.",
   i+7,"Time series of ",names(kdf1)[i]),xaxp=c(500,500,1),type="n")
          lines(1:nrow(kdf1),kdf1[,i])
          axis(1,c(50,200,400),  kdf1[c(50,200,400),"date"])                }

#Scatterplots infla vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(kdf1$infla24X,kdf1[,i],xlab="inflation",ylab=names(kdf1
   [i]), main=paste('Fig.',i+16,"Plot of",names(kdf1[i]),'vs inflation'))}
#Scatterplots amb vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(kdf1$amb,kdf1[,i],xlab="amb",ylab=names(kdf1[i]),
   main=paste('Fig.',i+25,"Plot of",names(kdf1[i]),'vs amb'))}
#Scatterplots m1 vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(kdf1$m1,kdf1[,i],xlab="m1",ylab=names(kdf1[i]),
   main=paste('Fig.',i+34,"Plot of",names(kdf1[i]),'vs m1'))}
#Scatterplots m2 vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(kdf1$m2,kdf1[,i],xlab="m2",ylab=names(kdf1[i]),
   main=paste('Fig.',i+43,"Plot of",names(kdf1[i]),'vs m2'))}

#Scatterplots log(amb) vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(log(kdf1$amb),kdf1[,i],xlab="amb",ylab=names(kdf1[i]),
   main=paste('Fig.',i+52,"Plot of",names(kdf1[i]),'vs log(amb)'))}
#Scatterplots log(m1) vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(log(kdf1$m1),kdf1[,i],xlab="m1",ylab=names(kdf1[i]),
   main=paste('Fig.',i+61,"Plot of",names(kdf1[i]),'vs log(m1)'))}
#Scatterplots log(m2) vs interest rates
par(mfcol=c(3,3)); for (i in
   7:15){scatter.smooth(log(kdf1$m2),kdf1[,i],xlab="m2",ylab=names(kdf1[i]),
   main=paste('Fig.',i+70,"Plot of",names(kdf1[i]),'vs log(m2)'))}
```

```
Y<-na.omit(kdfa)
names(Y)
y<-
   Y[,c("amb","m1","m2","m3","cpi","infla24X","tb3","tb6","irb1","irb2","irb5",
   "irb7","irb10","irb20","irb30","syc")]
round(cor(y),3)

##################################################################################
   ##
#Function medrpc
   medAbsPctResid<-function(myObj,linLog12){
            # 1 - linear, # 2 - log
            if (linLog12==1) {  a<-myObj$fitted.values+myObj$residuals
                                p<-myObj$fitted.values
                                r<-myObj$residuals  }
            if (linLog12==2) {  a<-exp(myObj$fitted.values+myObj$residuals)
                                p<-exp(myObj$fitted.values)
                                r<-a-p           }
               z<-median(abs(r/a)); return(z)          }

# Function iteration
rowNoResultsDF<-function(i,j,regID){z<-(j-1)*54+(i-1)*6+regID; return(z)}
rowNoResultsDF(1,1,6)
rowNoResultsDF(9,3,6)
rowNoResultsDF(9,2,1:6)
rowNoResultsDF(1:9,1:3,6)
for (j in 1:3){
   for (i in 1:9){
      for (z in 1:6){
      print(c(i,j,z,rowNoResultsDF(i,j,z)))
   }}}
##################################################################################
   ##


##################################################################################
   ##

gamResultsDF<-as.data.frame(matrix(nrow=100,ncol=10))
linResultsDF<-as.data.frame(matrix(nrow=100,ncol=17))
logMSResultsDF<-as.data.frame(matrix(nrow=100,ncol=17))
expResultsDF<-as.data.frame(matrix(nrow=100,ncol=19))
sLogResultsDF<-as.data.frame(matrix(nrow=100,ncol=19))
dLogResultsDF<-as.data.frame(matrix(nrow=100,ncol=20))

# invYCResults<-as.data.frame(matrix(nrow=100,ncol=3))

names(kdf)
instrumentVector<-names(kdf)[2:10]
moneySupplyVector<-names(kdf)[12:16]
instrumentVector
moneySupplyVector

min(kdf$tb3)
sort(kdf$tb3)
min(kdf$tb6)
kdf$tb3<-ifelse(kdf$tb3<0,.01,kdf$tb3)

for (jMS in 1:5){
   for (iIR in 1:9){
        # iIR<-4; jMS<-2
        # iIR<-1; jMS<-1
```

```
    print(c(iIR,jMS,rowNoResultsDF(i,j,1)))
    myIntRate<-instrumentVector[iIR]
    myMoneySupply<-moneySupplyVector[jMS]
    kdf$myVar<-kdf[,myIntRate]
    kdf$myInfla<-kdf$infla24X/2
    kdf$myInfla<-log(1+kdf$myInfla)
    names(kdf)
    tdf<-kdf[,c(myIntRate,myMoneySupply,"myInfla","date","cpi")]
    #!!!!!!! newLine Below  guessed input data for 2022.12
    tdf[nrow(kdf)+1,1:5]<-c(NA,5.20,.06,2022.12,305.0)
    names(tdf)[1:2]<-c("ir","ms")
    tdf$irLog<-log(tdf$ir)
    tdf$msLog<-log(tdf$ms)
    tdf$cpiLog<-log(tdf$cpi)
    n<-nrow(tdf)
    isdf<-tdf[tdf$date< 2022.07,]
    osdf<-tdf[tdf$date==2022.07,]
    osdf12<-tdf[tdf$date==2022.12,]  #NEW !!!!!!!
    actual<-osdf[1]
    isdf<-na.omit(isdf)
    invYCdf<-tdf[tdf$date==1980.03,]

    # gam - nonparametric regression
    gamFit<-gam(irLog~s(msLog,3)+s(myInfla,2)+s(cpiLog,1),data=isdf); #
par(mfcol=c(2,2)); plot.gam(gamFit)
       rpctGam<-medAbsPctResid(gamFit,1); rpct1;
       pred1Y<-predict.gam(gamFit,osdf12)
       #pred1980<-predict.gam(gamFit, invertYCdf)
       gamResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("gam",myMoneySupply,myIntRate)
       gamResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,exp(predict(gamFit,osdf)),exp(pred1Y))#
       gamResultsDF[rowNoResultsDF(iIR,jMS,1),8:11]<-
c(rpctGam,fRSq(gamFit,1),fMedAbsErr(gamFit),fMeanAbsAErr(gamFit))

    # basic lm model
    linFit<-lm(ir~ms+cpi+myInfla,data=isdf,na.action=na.omit);
summary(linFit)
       rpct1<-medAbsPctResid(linFit,1); rpct1;
       pred1Y<-predict(linFit,osdf12)
       predIVyc<-predict(linFit,invYCdf)
       zero<-
(coefficients(linFit)[1]+coefficients(linFit)[2]*(osdf$ms)+coefficients(linF
it)[2]*(osdf$cpi))
       linResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("lin",myMoneySupply,myIntRate)
       linResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,predict(linFit,osdf),exp(pred1Y))#
       linResultsDF[rowNoResultsDF(iIR,jMS,1),8:20]<-
c(rpct1,fRSq(linFit,1),fMedAbsErr(linFit),fMeanAbsAErr(linFit),zero,coeffici
ents(linFit),summary(linFit)$coefficients[,3])
       linResultsDF[rowNoResultsDF(iIR,jMS,1),21]<-predIVyc

    # using nls algorithm to estimate linear model with log transformation of
MS
       nlsFitLog<-nls(ir~a + bms*msLog+
binfla*myInfla+bcpi*cpi,data=isdf,start=list(a=2,bms=-2,binfla=68,bcpi=0));
# this nls has 3 parameters
       rpct1<-medAbsPctResid(nlsFitLog,1); rpct1;
       zero<-
(coefficients(nlsFitLog)[1]+coefficients(nlsFitLog)[2]*log(osdf$ms)+coeffici
ents(nlsFitLog)[4]*(osdf$cpi))
```

```
       pred1Y<-predict(nlsFitLog,osdf12)
       predIVyc<-predict(nlsFitLog,invYCdf)
       logMSResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("nlsFitLog",myMoneySupply,myIntRate)
       logMSResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,predict(nlsFitLog,osdf),exp(pred1Y)) #
       logMSResultsDF[rowNoResultsDF(iIR,jMS,1),8:20]<-
c(rpct1,fRSq(nlsFitLog,1),fMedAbsErr(nlsFitLog),fMeanAbsAErr(nlsFitLog),zero
,coefficients(nlsFitLog),summary(nlsFitLog)$parameters[,3])  # <= this is
the nls object
       logMSResultsDF[rowNoResultsDF(iIR,jMS,1),21]<-predIVyc

   # (nonLinear exponential) use nls algorithm to estimate non linear
# exponential model with exp(ems)
   nlsFitExp<-nls(ir~a + bms*2.718^(ems*ms) +
binfla*myInfla+bcpi*cpi,data=isdf,start=list(a=5,bms=2,ems=-
.5,binfla=68,bcpi=.002),nls.control(maxiter=1000));   summary(nlsFitExp)

       rpct1<-medAbsPctResid(nlsFitExp,1); rpct1;
       zero<-
coefficients(dLogFit)[1]+coefficients(nlsFitExp)[2]*exp(coefficients(nlsFitE
xp)[3]*osdf$ms)+coefficients(nlsFitExp)[5]*(osdf$cpi)
       pred1Y<-predict(nlsFitExp,osdf12)
       predIVyc<-predict(nlsFitExp,invYCdf)          ###NEW !!!!!
       expResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("nlsFitExp",myMoneySupply,myIntRate)
       expResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,predict(nlsFitExp,osdf),(pred1Y))#
       expResultsDF[rowNoResultsDF(iIR,jMS,1),8:22]<-
c(rpct1,fRSq(nlsFitExp,1),fMedAbsErr(nlsFitExp),fMeanAbsAErr(nlsFitExp),zero
,coefficients(nlsFitExp),summary(nlsFitExp)$parameters[,3])  # <= this is
the nls object
       expResultsDF[rowNoResultsDF(iIR,jMS,1),23]<-predIVyc

   sLogFit<-lm(irLog~ms+cpi+myInfla,data=isdf,na.action=na.omit);
summary(sLogFit)
       p<-exp(sLogFit$fitted.values)
       cor(isdf$ir,p)^2
       zero<-
exp(coefficients(sLogFit)[1]+coefficients(sLogFit)[2]*osdf$ms+coefficients(s
LogFit)[3]*osdf$cpi)
       rpct1<-medAbsPctResid(sLogFit,2); rpct1;
       pred1Y<-predict(sLogFit,osdf12)
       predIVyc<-exp(predict(sLogFit,invYCdf))         ###NEW !!!!!
       sLogResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("sLog",myMoneySupply,myIntRate)
       sLogResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,exp(predict(sLogFit,osdf)),exp(pred1Y))#
       sLogResultsDF[rowNoResultsDF(iIR,jMS,1),8:20]<-
c(rpct1,fRSq(sLogFit,1),fMedAbsErr(sLogFit),fMeanAbsAErr(sLogFit),zero,coeff
icients(sLogFit),summary(sLogFit)$coefficients[,3])
       sLogResultsDF[rowNoResultsDF(iIR,jMS,1),21]<-predIVyc


       dLogFit<-lm(irLog~msLog+log(cpi)+log(1+myInfla),data=isdf);
summary(dLogFit)
       p<-exp(dLogFit$fitted.values)
       zero<-
exp(coefficients(dLogFit)[1]+coefficients(dLogFit)[2]*log(osdf$ms)+coefficie
nts(dLogFit)[3]*log(osdf$cpi))
       cor(isdf$ir,p)^2
       rpct1<-medAbsPctResid(dLogFit,1); rpct1;
       pred1Y<-predict(dLogFit,osdf12)
```

```
        predIVyc<- exp( predict(dLogFit,invYCdf))          ###NEW !!!!!
        dLogResultsDF[rowNoResultsDF(iIR,jMS,1),1:3]<-
c("dLogFit",myMoneySupply,myIntRate)
        dLogResultsDF[rowNoResultsDF(iIR,jMS,1),4:7]<-
c(n,actual,exp(predict(dLogFit,osdf)),exp(pred1Y))#
        dLogResultsDF[rowNoResultsDF(iIR,jMS,1),8:20]<-
c(rpct1,fRSq(dLogFit,1),fMedAbsErr(dLogFit),fMeanAbsAErr(dLogFit),zero,coeff
icients(dLogFit),summary(dLogFit)$coefficients[,3])
        dLogResultsDF[rowNoResultsDF(iIR,jMS,1),21]<-predIVyc

        }}

    dim(gamResultsDF); dim(linResultsDF); dim(logMSResultsDF);
dim(sLogResultsDF); dim(dLogResultsDF); dim(expResultsDF)

    names(gamResultsDF)<-
c("regType",'MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr')
    names(linResultsDF)<-
c('regType','MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr',"zeroInfla",'int','bMS',"bcpi",'bInlfa','tI
nt','tMS',"tcpi",'tInlfa',"Pred1980")

    names(logMSResultsDF)<-
c('regType','MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr',"zeroInfla",'int','bMS',"bcpi",'bInlfa','tI
nt','tMS',"tcpi",'tInlfa',"Pred1980")
    names(sLogResultsDF)<-
c('regType','MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr',"zeroInfla",'int','bMS',"bcpi",'bInlfa','tI
nt','tMS',"tcpi",'tInlfa',"Pred1980")
    names(dLogResultsDF)<-
c('regType','MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr',"zeroInfla",'int','bMS',"bcpi",'bInlfa','tI
nt','tMS',"tcpi",'tInlfa',"Pred1980")
    names(expResultsDF)<-
c('regType','MoneySupply','Instrument','n','ActualVal','PredVal','PredVal1',
'rpct','R2','MedAErr','MeanAErr',"zeroInfla",'int','bMS',"ems","bcpi",'bInlf
a','tInt','tMS',"tms","tcpi",'tInlfa','Pred1980')

    linResultsDF
    logMSResultsDF
    sLogResultsDF
    dLogResultsDF
    expResultsDF

    gamResultsDF<-na.omit(gamResultsDF); linResultsDF<-na.omit(linResultsDF);
logMSResultsDF<-na.omit(logMSResultsDF); expResultsDF<-
na.omit(expResultsDF); sLogResultsDF<-na.omit(sLogResultsDF);
dLogResultsDF<-na.omit(dLogResultsDF)
    dim(gamResultsDF); dim(linResultsDF); dim(logMSResultsDF);
dim(sLogResultsDF); dim(dLogResultsDF);     dim(expResultsDF)

    linResultsDF[,6:21]<-round(linResultsDF[,6:21],2)
    logMSResultsDF[,6:21]<-round(logMSResultsDF[,6:21],2)
    expResultsDF[,6:23]<-round(expResultsDF[,6:23],2)
    sLogResultsDF[,6:21]<-round(sLogResultsDF[,6:21],2)
    dLogResultsDF[,6:21]<-round(dLogResultsDF[,6:21],2)

r2PT<-as.data.frame(round(rbind(
    tapply(gamResultsDF$R2, gamResultsDF$MoneySupply,median), # but wrong,
    tapply(linResultsDF$R2, linResultsDF$MoneySupply,median),
    tapply(logMSResultsDF$R2, logMSResultsDF$MoneySupply,median),
```

```
    tapply(expResultsDF$R2, expResultsDF$MoneySupply,median),
    tapply(sLogResultsDF$R2, sLogResultsDF$MoneySupply,median),
    tapply(dLogResultsDF$R2, dLogResultsDF$MoneySupply,median))
    ,3))
row.names(r2PT)<-c("gam","basic","logMS","emsNExp","sLog","dLog")
r2PT

r2PTmean<-as.data.frame(round(rbind(
    tapply(gamResultsDF$R2, gamResultsDF$MoneySupply,mean), # but wrong,
    tapply(linResultsDF$R2, linResultsDF$MoneySupply,mean),
    tapply(logMSResultsDF$R2, logMSResultsDF$MoneySupply,mean),
    tapply(expResultsDF$R2, expResultsDF$MoneySupply,mean),
    tapply(sLogResultsDF$R2, sLogResultsDF$MoneySupply,mean),
    tapply(dLogResultsDF$R2, dLogResultsDF$MoneySupply,mean))
    ,3))
row.names(r2PTmean)<-c("gam","basic","logMS","emsNExp","sLog","dLog")
r2PTmean

medAbsErrPT<-as.data.frame(round(rbind(
    tapply(gamResultsDF$MedAErr, gamResultsDF$MoneySupply,median), # but
wrong,
    tapply(linResultsDF$MedAErr, linResultsDF$MoneySupply,median),
    tapply(logMSResultsDF$MedAErr,logMSResultsDF$MoneySupply,median),
    tapply(expResultsDF$MedAErr, expResultsDF$MoneySupply,median),
    tapply(sLogResultsDF$MedAErr, sLogResultsDF$MoneySupply,median),
    tapply(dLogResultsDF$MedAErr, dLogResultsDF$MoneySupply,median))
    ,3))
row.names(medAbsErrPT)<-c("gam","basic","logMS","emsNExp","sLog","dLog")
#best dLog+m2
medAbsErrPT

meanAbsErrPT<-as.data.frame(round(rbind(
    tapply(gamResultsDF$MeanAErr, gamResultsDF$MoneySupply,mean), # but
wrong,
    tapply(linResultsDF$MeanAErr, linResultsDF$MoneySupply,mean),
    tapply(logMSResultsDF$MedAErr,logMSResultsDF$MoneySupply,mean),
    tapply(expResultsDF$MeanAErr, expResultsDF$MoneySupply,mean),
    tapply(sLogResultsDF$MeanAErr, sLogResultsDF$MoneySupply,mean),
    tapply(dLogResultsDF$MeanAErr, dLogResultsDF$MoneySupply,mean))
    ,3))
row.names(meanAbsErrPT)<-c("gam","basic","logMS","emsNExp","sLog","dLog")
#best dLog+m2
meanAbsErrPT

r2PT
r2PTmean
medAbsErrPT
meanAbsErrPT

dLogResultsDF[dLogResultsDF$MoneySupply=='amb',]
expResultsDF[expResultsDF$MoneySupply=="amb",]



DF<-dLogResultsDF[dLogResultsDF$MoneySupply=="m2",]
DF$Actual1980<-c( 14.01,13.99,15.45,14.73,13.48,13.1,12.72,12.12,12.25)
DF
DF1<-dLogResultsDF[dLogResultsDF$MoneySupply=="amb",]
DF1$Actual1980<-c( 14.01,13.99,15.45,14.73,13.48,13.1,12.72,12.12,12.25)



DF[,"1980"]
```

```
   #names(kdf)
   kdf[kdf$date==1980.02,2:10]   #act values sto 1980.02
DF$Actual1980<-c( 14.01,13.99,15.45,14.73,13.48,13.1,12.72,12.12,12.25)

############################################################

ActualVal<-DF1$ActualVal
PredVal<-DF1$PredVal
PredVal1<-DF1$PredVal1
ZeroInfla<-DF1$zeroInfla
maturityVector<-c(.25,.5,1,2,5,7,10,20,30)
plot(maturityVector,unlist(ActualVal),xlim=c(0,33),ylim=c(0,5),type="n",xlab="m
   aturity",ylab="YTM", main= "Yield Curves")
 lines(maturityVector,unlist(ActualVal),col="red",lty=1,lwd=3);
 lines(maturityVector,unlist(PredVal),col="blue",lty=1,lwd=3);
 lines(maturityVector,unlist(PredVal1),col="green",lty=1,lwd=3);
 lines(maturityVector, unlist(ZeroInfla),col="orange",lty=1,lwd=3)
 legend(15,5,c("Actual Values 02/2022", "Pred Values 02/2022","Zero Inflation
   02/2022"),lty=c(1,1,1,1),col=c("red","blue","orange"),lwd=3)

 legend(15,5,c("Actual Values 02/2022", "Pred Values 02/2022", "Pred Values
   12/2022", "Zero Inflation
   02/2022"),lty=c(1,1,1,1),col=c("red","blue","green","orange"),lwd=3)
# col=c("red","blue","green","black")

colors()
length(colors())
data.frame(colName=colors(),colNo=1:147)
tsplot(kdfa$myVar,col=120)
tsplot(kdfa$myVar,col="red")
par(mfcol=c(7,7))
for (i in 1:147){tsplot(kdfa$myVar,col=i)}

ActualVal<-DF1$Actual1980
PredVal<-DF1$Pred1980
maturityVector<-c(.25,.5,1,2,5,7,10,20,30)
plot(maturityVector,unlist(ActualVal),xlim=c(0,33),ylim=c(0,17),type="n",xlab="
   maturity",ylab="YTM", main= "1980 Yield Curves")
 lines(maturityVector,unlist(ActualVal),col="red",lty=1,lwd=3);
 lines(maturityVector,unlist(PredVal),col="blue",lty=1,lwd=3);

 legend(15,5,c("Actual Values 03/1980", "Pred Values
   03/1980"),lty=c(1,1,1,1),col=c("red","blue"),lwd=3)
# col=c("red","blue","green","black")

###############################################################################
   ##########

par(mfcol=c(3,3));
for (iIR in 1:9){
        # iIR<-4; jMS<-2
         # iIR<-1; jMS<-1

       myIntRate<-instrumentVector[iIR]

       kdf$myVar<-kdf[,myIntRate]
       kdf$myInfla<-kdf$infla24X/2
       kdf$myInfla<-log(1+kdf$myInfla)
       names(kdf)
       newdf<-kdf[,c(myIntRate,"amb","myInfla","date","cpi")]
       #!!!!!!! newLine Below guessed input data for 2022.12
       newdf[nrow(kdf)+1,1:5]<-c(NA,5.20,.06,2022.12,305.0)
       names(newdf)[1:2]<-c("ir","amb")
```

```
        newdf$irLog<-log(newdf$ir)
        #tdf$msLog<-log(tdf$ms)
        #tdf$cpiLog<-log(tdf$cpi)
        n<-nrow(newdf)
        isdf<-newdf[newdf$date< 2022.07,]
        osdf<-newdf[newdf$date==2022.07,]
        osdf12<-newdf[newdf$date==2022.12,]  #NEW !!!!!!!
        actual<-osdf[1]
        isdf<-na.omit(isdf)


        #dLogFit<-lm(irLog~log(amb)+log(cpi)+log(1+myInfla),data=isdf);
    summary(dLogFit)
        #res <- resid(dLogFit)
        msFitExp<-ms(~(ir-(a + bms*2.718^(ems*amb) +
    binfla*myInfla+bcpi*cpi))^2,data=isdf,start=list(a=13,bms=-2,ems=-
    1.2,binfla=28,bcpi=-0.03),nls.control(maxiter=1000));   #summary(nlsFitExp)

        fcoef(msFitExp)
        fdataframe(isdf)

        pred<-a + bms*2.718^(ems*amb) + binfla*myInfla+bcpi*cpi
        r<-ir-pred

         plot(pred, r, main= paste("Fig.",iIR+114,"Residuals
    for",myIntRate),xlab="Predicted Values" ,ylab="Residuals"); abline(0,0)
        }

par(mfcol=c(3,3));
for (iIR in 1:9){
       # iIR<-4
         # iIR<-1; jMS<-1

        myIntRate<-instrumentVector[iIR]

        kdf$myVar<-kdf[,myIntRate]
        kdf$myInfla<-kdf$infla24X/2
        kdf$myInfla<-log(1+kdf$myInfla)
        names(kdf)
        newdf<-kdf[,c(myIntRate,"amb","myInfla","date","cpi")]
        #!!!!!!! newLine Below  guessed input data for 2022.12
        newdf[nrow(kdf)+1,1:5]<-c(NA,5.20,.06,2022.12,305.0)
        names(newdf)[1:2]<-c("ir","amb")
        newdf$irLog<-log(newdf$ir)
        #tdf$msLog<-log(tdf$ms)
        #tdf$cpiLog<-log(tdf$cpi)
        n<-nrow(newdf)
        isdf<-newdf[newdf$date< 2022.07,]
        osdf<-newdf[newdf$date==2022.07,]
        osdf12<-newdf[newdf$date==2022.12,]  #NEW !!!!!!!
        actual<-osdf[1]
        isdf<-na.omit(isdf)


        #dLogFit<-lm(irLog~log(amb)+log(cpi)+log(1+myInfla),data=isdf);
    summary(dLogFit)
        #res <- resid(dLogFit)
        #qqnorm(res,main=paste("Fig.",iIR+96,"Normal QQ plot
    for",myIntRate),ylab="Residuals");qqline(res)
```

```
    #nlsFitExp<-nls(ir~a + bms*2.718^(ems*amb) +
binfla*myInfla+bcpi*cpi,data=isdf,start=list(a=13,bms=-2,ems=-
1.2,binfla=28,bcpi=-0.03),nls.control(maxiter=1000));   #summary(nlsFitExp)
    msFitExp<-ms(~(ir-(a + bms*2.718^(ems*amb) +
binfla*myInfla+bcpi*cpi))^2,data=isdf,start=list(a=13,bms=-2,ems=-
1.2,binfla=28,bcpi=-0.03),nls.control(maxiter=1000));   #summary(nlsFitExp)

    fcoef(msFitExp)
    fdataframe(isdf)

    pred<-a + bms*2.718^(ems*amb) + binfla*myInfla+bcpi*cpi
    r<-ir-pred

    #Time Series plot of residuals

    #tsplot(r,xlab="Year/Month",xaxt="n",ylab="Residuals",main=paste("Fig.",i
IR,"TSplot of Residuals",myIntRate)); abline(h=0,lty=3)
        #axis(1,c(50,200,400),  kdf1[c(50,200,400),"date"])


    #Time Series plots of actual and predicted
        tsplot(kdf[,myIntRate],pred,xlab="Year/Month",xaxt="n",ylab="Interest
Rates",main=paste("Fig.",iIR,"TSplot of Pred & Actual",myIntRate))
        legend(150, 15, legend=c("Actual Values", "Predicted Values"),
lty=1:2,cex=0.4)
        axis(1,c(50,200,400),  kdf1[c(50,200,400),"date"])


#length(pred)
#nrow(kdf1)

    #QQplot of residuals
    #res <- resid(()
    #qqnorm(r,main=paste("Fig.",iIR+123,"Normal QQ plot
for",myIntRate),ylab="Residuals");qqline(res)

}

par(mfcol=c(3,3));
for (iIR in 1:9){
     # iIR<-4; jMS<-2
     # iIR<-1; jMS<-1

    myIntRate<-instrumentVector[iIR]

    kdf$myVar<-kdf[,myIntRate]
    kdf$myInfla<-kdf$infla24X/2
    kdf$myInfla<-log(1+kdf$myInfla)
    names(kdf)
    newdf<-kdf[,c(myIntRate,"amb","myInfla","date","cpi")]
    #!!!!!!! newLine Below  guessed input data for 2022.12
    newdf[nrow(kdf)+1,1:5]<-c(NA,5.20,.06,2022.12,305.0)
    names(newdf)[1:2]<-c("ir","amb")
    newdf$irLog<-log(newdf$ir)
    #tdf$msLog<-log(tdf$ms)
    #tdf$cpiLog<-log(tdf$cpi)
    n<-nrow(newdf)
    isdf<-newdf[newdf$date< 2022.07,]
    osdf<-newdf[newdf$date==2022.07,]
    osdf12<-newdf[newdf$date==2022.12,]  #NEW !!!!!!!
    actual<-osdf[1]
    isdf<-na.omit(isdf)
```

```
    #dLogFit<-lm(irLog~log(amb)+log(cpi)+log(1+myInfla),data=isdf);
summary(dLogFit)
    #res <- resid(dLogFit)

    msFitExp<-ms(~(ir-(a + bms*2.718^(ems*amb) +
binfla*myInfla+bcpi*cpi))^2,data=isdf,start=list(a=13,bms=-2,ems=-
1.2,binfla=28,bcpi=-0.03),nls.control(maxiter=1000));   #summary(nlsFitExp)

    fcoef(msFitExp)
    fdataframe(isdf)

    pred<-a + bms*2.718^(ems*amb) + binfla*myInfla+bcpi*cpi
    r<-ir-pred

    plot(density(r),main=paste("Fig.",iIR+132,"Density of
Residuals",myIntRate),ylab="Density",xlab="Residuals",type="l")}


##############################################################################
    ##############################################

xVec<-1:1000
yVec<-seq(.01,10,.01)
plot(xVec,yVec)
dVec<-8-.01*xVec
length(dVec)

plot(xVec,yVec,type="n", main="Fig. 3 Money Supply and Demand",ylab="interest
    rate",xlab="Money");
        lines(xVec,dVec,, col='blue')
        abline(v=400,col='red');    abline(v=500, lty=3, col='red')
equilibVecy<-rep(4,400); equilibVecx<-1:400;
    lines(equilibVecx,equilibVecy,lty=2)
equilibVecy2<-rep(3,500); equilibVecx2<-1:500; lines(equilibVecx2,equilibVecy2,
    lty=2)
text(440,8,"MS0")
text(540,8,"MS1")
text(800,1,"MD")

xVec<-1:1000
yVec<-seq(.01,10,.01)
plot(xVec,yVec)
dVec<-8-.01*xVec
length(dVec)

plot(xVec,yVec,type="n", main="Fig. 4 Money Supply and Demand",ylab="interest
    rate",xlab="Money");
        lines(xVec,dVec, col='blue')
        abline(v=400,col='red');    abline(v=300, lty=3,col='red')
equilibVecy<-rep(4,400); equilibVecx<-1:400; lines(equilibVecx,equilibVecy,
    lty=2)
equilibVecy2<-rep(5,300); equilibVecx2<-1:300;
    lines(equilibVecx2,equilibVecy2,lty=2)
text(440,8,"MS0")
text(340,8,"MS1")
text(800,1,"MD")



xVec<-1:1000
yVec<-seq(.01,10,.01)
```

```
plot(xVec,yVec)
dVec<-8-.01*xVec
uVec<-10-0.01*xVec
length(uVec)
length(dVec)

plot(xVec,yVec,type="n", main="Fig. 3 Money Supply and Demand",ylab="interest
    rate",xlab="Money");
      lines(xVec,dVec, col='blue')
       lines(xVec,dVec, col='blue',lty=3)


matplot(xVec,
    cbind(dVec,uVec),type="l",col=c("blue","blue"),lty=c(1,3),main="Fig. 5 Money
    Supply and Demand",ylab="interest rate",xlab="Money")
     abline(v=400,col='red');
equilibVecy<-rep(4,400); equilibVecx<-1:400; lines(equilibVecx,equilibVecy,
    lty=2)
equilibVecy<-rep(6,400); equilibVecx<-1:400; lines(equilibVecx,equilibVecy,
    lty=2)

text(440,8,"MS")
text(800,1,"MD0")
text(800,3,"MD1")


uVec<-6-0.01*xVec

matplot(xVec,
    cbind(dVec,uVec),type="l",col=c("blue","blue"),lty=c(1,3),main="Fig. 6 Money
    Supply and Demand",ylab="interest rate",xlab="Money")
     abline(v=400,col='red');
equilibVecy<-rep(4,400); equilibVecx<-1:400; lines(equilibVecx,equilibVecy,
    lty=2)
equilibVecy<-rep(2,400); equilibVecx<-1:400; lines(equilibVecx,equilibVecy,
    lty=2)
text(440,8,"MS")
text(800,1,"MD0")
text(800,-1,"MD1")
```

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